

Answer on Question #59275 – Math – Statistics and Probability

Question

A survey was conducted to measure the number of hours per week adults spend on home computers. In the survey, the number of hours were normally distributed, with a mean of 7 hours and a standard deviation of 1.5 hours.

- a) What is the probability that the sample mean of 9 participants exceeds 8 hours?
- b) What is the probability that the sample mean of 9 participants is below 6.5 hours?
- c) What is the probability that the sample mean of 25 participants is between 6.8 and 7.8 hours?

Solution

First of all, we note that if the random variables $\xi_1, \xi_2, \dots, \xi_n$ are normally distributed, with a mean of a and a standard deviation of σ then their mean $\bar{\xi} := \frac{1}{n} \sum_{k=1}^n \xi_k$ is normally distributed, with a mean of a and a standard deviation of $\frac{\sigma}{\sqrt{n}}$.

Note also that

$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{u^2}{2}} du$ is the function of Laplace.

a) In this case $n = 9$, $\sigma_n = \frac{\sigma}{\sqrt{n}} = \frac{1.5}{\sqrt{9}} = \frac{1.5}{3} = 0.5$, $\bar{\xi} \sim N(7, 0.5)$. Then

$$P\{\bar{\xi} > 8\} = P\left\{\frac{\bar{\xi}-7}{0.5} > \frac{8-7}{0.5}\right\} = P\left\{\frac{\bar{\xi}-7}{0.5} > 2\right\} = 0.5 - \Phi(2) = \left\{\begin{array}{l} \text{from the table} \\ \text{of Laplace} \end{array}\right\} = 0.5 - 0.47725 = 0.02275 \approx 0.0228.$$

b) Similarly to a) we have

$$\begin{aligned} P\{\bar{\xi} < 6.5\} &= P\left\{\frac{\bar{\xi}-7}{0.5} < \frac{6.5-7}{0.5}\right\} = P\left\{\frac{\bar{\xi}-7}{0.5} < -1\right\} = 0.5 - \Phi(1) = 0.5 - 0.34134 = \\ &= 0.15866 \approx 0.1587. \end{aligned}$$

c) In this case $n = 25$, $\sigma_n = \frac{\sigma}{\sqrt{n}} = \frac{1.5}{\sqrt{25}} = \frac{1.5}{5} = 0.3$, $\bar{\xi} \sim N(7, 0.3)$. Then

$$P\{6.8 < \bar{\xi} < 7.8\} = P\left\{\frac{6.8-7}{0.3} < \frac{\bar{\xi}-7}{0.3} < \frac{7.8-7}{0.3}\right\} = P\left\{-0.67 < \frac{\bar{\xi}-7}{0.3} < 2.67\right\} = \Phi(2.67) + \Phi(0.67) = 0.49621 + 0.24857 = 0.74478 \approx 0.7448.$$

Answer: a) 0.0228; b) 0.1587; c) 0.7448.