

Answer on Question #59251 – Math – Statistics and Probability

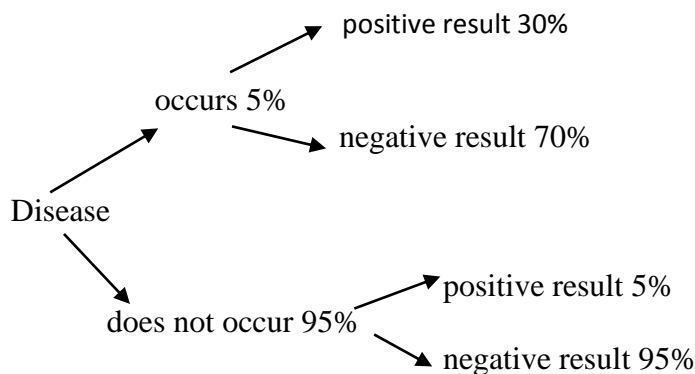
Question

A certain medical disease occurs in 5% of the population. A simple screening procedure is available and in 3 out of 10 cases where the patient has the disease it provides a positive result. If the patient does not have the disease there is still a 0.05 chance that the test will give a positive result. Draw a tree diagram to represent this.

- a) Find the probability that a randomly selected individual does not have the disease but gives a positive result in the screening test gives a positive result on the test.
- b) Ben has taken the test and his result is positive. Find the probability that he has the disease.

Solution

A tree diagram is given below.



Introduce H_1 = "disease occurs", H_2 = "disease does not occur", D = "positive result",

$$P(H_1) = 0.05, P(H_2) = 0.95, P(D|H_1) = 0.3, P(D|H_2) = 0.05.$$

- a) By the rule of multiplication, the probability that a randomly selected individual does not have the disease but gives a positive result in the screening test is

$$P_1 = P(H_2 \cap D) = P(D|H_2)P(H_2) = 0.05 \cdot 0.95 = 0.0475 \text{ or } 4.75\%.$$

- b) We shall apply Bayes' Theorem below.

Given his result is positive the probability that he has the disease is

$$P(H_1|D) = \frac{P(D|H_1)P(H_1)}{P(D)} = \frac{P(D|H_1)P(H_1)}{P(D|H_1)P(H_1) + P(D|H_2)P(H_2)} = \frac{0.3 \cdot 0.05}{0.3 \cdot 0.05 + 0.05 \cdot 0.95} = 0.24 \text{ or } 24\%.$$

Answer: a) 0.0475; b) 0.24.