Answer on Question #59208 – Math – Calculus

Question

5. Given that φ=2x^2y−xz^3 find ∇^2φ

Solution

$$\varphi = 2x^2y - xz^3$$

$$\nabla^2\varphi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\varphi$$

$$\frac{\partial^2}{\partial x^2}\varphi = 4y, \quad \frac{\partial^2}{\partial y^2}\varphi = 0, \quad \frac{\partial^2}{\partial z^2}\varphi = -6zx$$

$$\nabla^2\varphi = 4y - 6xz$$

Answer:

$$\nabla^2\varphi = 4y - 6xz$$

Question

6. If A=xz^3i−2x^2yzj+2yz^4k, find ∇×A at point (1,-1,1).

Solution

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = i\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - j\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + k\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) =$$

$$= i(2z^4 + 2x^2y) - j(-3xz^2) + k(-4xyz) = 2i(z^4 + x^2y) + 3xz^2j - 4xyzk$$

$$\nabla \times A(1,-1,1) = 2i(1-1) + 3j + 4k = 3j + 4k = (0,3,4)$$

Answer:

$$\nabla \times A(1,-1,1) = 3j + 4k = (0,3,4)$$

Question

7. Given that A=A1i+A2j+A3k and r=xi+yj+zk, evaluate ∇ · (A×r) if ∇×A=0

Solution

$$A \times r = \begin{vmatrix} i & j & k \\ x & y & z \\ A_1 & A_2 & A_3 \end{vmatrix} = i(yA_3 - zA_2) - j(xA_3 - zA_1) + k(xA_2 - yA_1)$$

$$\nabla \cdot (\mathrm{A} \times \mathrm{r}) = \frac{\partial (A \times r)_x}{\partial x} + \frac{\partial (A \times r)_y}{\partial y} + \frac{\partial (A \times r)_z}{\partial z} =$$

$$= \frac{\partial (yA_3 - zA_2)}{\partial x} + \frac{\partial (-xA_3 + zA_1)}{\partial y} + \frac{\partial (xA_2 - yA_1)}{\partial z}$$

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix} = i\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) - j\left(\frac{\partial A_3}{\partial x} - \frac{\partial A_1}{\partial z}\right) + k\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right) = 0$$

$$\frac{\partial (yA_3 - zA_2)}{\partial x} + \frac{\partial (-xA_3 + zA_1)}{\partial y} + \frac{\partial (xA_2 - yA_1)}{\partial z} =$$

$$= y\frac{\partial A_3}{\partial x} - y\frac{\partial A_1}{\partial z} - z\frac{\partial A_2}{\partial x} + z\frac{\partial A_1}{\partial y} - x\frac{\partial A_3}{\partial y} + x\frac{\partial A_2}{\partial z} = 0$$

Answer:
$\nabla \cdot (\mathrm{A} \times \mathrm{r}) = 0$

Question

8. Let A=x^2yi−2xzj+2yzk, find Curl curl A

Solution

$$curl\ A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = i\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - j\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + k\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) =$$

$$= i(2z + 2x) - j(0) + k(-2z - x^2)$$

$$curl\ curl\ A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (curlA)_x & (curlA)_y & (curlA)_z \end{vmatrix} =$$

$$= i\left(\frac{\partial (curlA)_z}{\partial y} - \frac{\partial A(curlA)_y}{\partial z}\right) - j\left(\frac{\partial (curlA)_z}{\partial x} - \frac{\partial (curlA)_x}{\partial z}\right) + k\left(\frac{\partial (curlA)_y}{\partial x} - \frac{\partial (curlA)_x}{\partial y}\right)$$

$$curl\ curl\ A = i(0 - 0) - j(-2x - 2) + k(0 - 0) = 2j(x - 1)$$

Answer:
$curl\ curl\ A = 2j(x - 1)$

Question

9. Given A=2x^2i−3yzj+xz^2k and φ=2z−x^3y, find A·∇φ at point (1,-1,1).

Solution

$$\nabla\varphi = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} = -3x^2yi - x^3j + 2k$$

$$A\nabla\varphi = -6x^4y + 3x^3yz + 2xz^2$$

Answer:
$A\nabla\varphi = -6x^4y + 3x^3yz + 2xz^2$

Question

10. Find the directional derivative of φ=x^2yz+4xz^2 at (1,-2,-1) in the direction 2i−j−2k

Solution

Let $\vec{l} = 2\vec{i} + \vec{j} - 2\vec{k} \to |\vec{l}| = \sqrt{4 + 1 + 4} = 3$

$A = (1, -2, -1)$

So, the direction cosines are

$$\cos\alpha = \frac{2}{3}, \cos\beta = \frac{1}{3}, \cos\gamma = -\frac{2}{3}$$

$$\frac{\partial\varphi}{\partial x}(A) = (2xyz + 4z^2)(A) = 4 + 4 = 8;$$

$$\frac{\partial\varphi}{\partial y}(A) = (x^2z)(A) = -1$$

$$\frac{\partial\varphi}{\partial z}(A) = (x^2y + 8xz)(A) = -2 - 8 = -10$$
$$\frac{\partial\varphi}{\partial l}(A) = \frac{\partial\varphi}{\partial x}(A)\cos\alpha + \frac{\partial\varphi}{\partial y}(A)\cos\beta + \frac{\partial\varphi}{\partial z}(A)\cos\gamma = 8 * \frac{2}{3} - 1 * \frac{1}{3} - 10 * \frac{2}{3} = \frac{16}{3} - \frac{1}{3} - \frac{20}{3}$$
$$\frac{\partial\varphi}{\partial l}(A) = -\frac{5}{3}$$

Answer:

$$-\frac{5}{3}$$