Answer on Question #59204 – Math – Linear Algebra

Question

6. If $A = 2i - 3j - k$ and $B = i + 4j - 2k$, find $(A + B +\times (A - B)$

Solution

Remark: Statement ***may contain a bug*** *and perhaps we have to find* $(A + B) \times (A - B)$.
If $A = 2i - 3j - k$ and $B = i + 4j - 2k$ then

$$(A + B) = 3i + j - 3k$$

and

$$(A - B) = i - 7j + k.$$

So we have the following cross product:

$$(A + B) \times (A - B) = \begin{vmatrix} i & j & k \\ 3 & 1 & -3 \\ 1 & -7 & 1 \end{vmatrix} = i \begin{vmatrix} 1 & -3 \\ -7 & 1 \end{vmatrix} - j \begin{vmatrix} 3 & -3 \\ 1 & 1 \end{vmatrix} + k \begin{vmatrix} 3 & 1 \\ 1 & -7 \end{vmatrix}$$

$$= (1 - 21)i - (3 + 3)j + (-21 - 1)k = -20i - 6j - 22k.$$

Answer: $-20i - 6j - 22k$.

Question

7. If $A = 3i - j + 2k$, $B = 2i + j - k$ and $C = i - 2j + 2k$, find $(A \times B) \times C$

Solution

If $A = 3i - j + 2k$, $B = 2i + j - k$ and $C = i - 2j + 2k$ then the cross product is

$$(A \times B) = \begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix} = i \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} - j \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} + k \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= (1 - 2)i - (-3 - 4)j + (3 + 2)k = -i + 7j + 5k$$

and the expression of the vector triple product

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -1 & 7 & 5 \\ 1 & -2 & 2 \end{vmatrix} = i \begin{vmatrix} 7 & 5 \\ -2 & 2 \end{vmatrix} - j \begin{vmatrix} -1 & 5 \\ 1 & 2 \end{vmatrix} + k \begin{vmatrix} -1 & 7 \\ 1 & -2 \end{vmatrix}$$

$$= (14 + 10)i - (-2 - 5)j + (2 - 7)k = 24i + 7j - 5k$$

Answer: $24i + 7j - 5k$.

Question

8. Determine a unit vector perpendicular to the plane of $A = 2i - 6j - 3k$ and $B = 4i + 3j - k$.

Solution

If $A = 2i - 6j - 3k$ and $B = 4i + 3j - k$ then the cross product is

$$A \times B = \begin{vmatrix} i & j & k \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix} = i \begin{vmatrix} -6 & -3 \\ 3 & -1 \end{vmatrix} - j \begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} + k \begin{vmatrix} 2 & -6 \\ 4 & 3 \end{vmatrix}$$

$$= (6 + 9)i - (-2 + 12)j + (6 + 24)k = 15i - 10j + 30k$$

and the unit vector perpendicular to the plane of A and B is

$$n = \frac{A \times B}{|A \times B|} = \frac{15i - 10j + 30k}{\sqrt{15^2 + 10^2 + 30^2}} = \frac{15i - 10j + 30k}{\sqrt{1225}} = \frac{15i - 10j + 30k}{35} = \frac{3i}{7} - \frac{2j}{7} + \frac{6k}{7}$$

Answer: $\frac{3}{7}i - \frac{2}{7}j + \frac{6}{7}k.$

Question

9. Evaluate $(2i - 3j) \cdot [(i + j - k) \times (3i - k)]$.

Solution

The cross product is

$$(i + j - k) \times (3i - k) = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix} = i\begin{vmatrix} 1 & -1 \\ 0 & -1 \end{vmatrix} - j\begin{vmatrix} 1 & -1 \\ 3 & -1 \end{vmatrix} + k\begin{vmatrix} 1 & 1 \\ 3 & 0 \end{vmatrix}$$

$$= (-1 - 0)i - (-1 + 3)j + (0 - 3)k = -i - 2j - 3k.$$

The scalar triple product is

$$(2i - 3j) \cdot [(i + j - k) \times (3i - k)] = (2i - 3j) \cdot (-i - 2j - 3k)$$

$$= 2 \cdot (-1) + (-3) \cdot (-2) + 0 \cdot (-3) = -2 + 6 = 4$$

Answer: 4.

Question

10. If $A = i - 2j - 3k$, $B = 2i + j - k$ and $C = i + 3j - 2k$, evaluate $(A \times B) \cdot C$.

Solution

The scalar triple product is

$$(A \times B) \cdot C = \begin{vmatrix} 1 & -2 & -3 \\ 2 & 1 & -1 \\ 1 & 3 & -2 \end{vmatrix} = 1\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} - (-2)\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} + (-3)\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}$$

$$= (-2 + 3) + 2(-4 + 1) - 3(6 - 1) = 1 - 6 - 15 = -20.$$

Answer: -20.
