Answer on Question #59203 – Math – Linear Algebra

If **u** and **v** are nonzero vectors (in R^2 *or* R^3) and if θ is the angle between **u** and **v,** then the dot product of **u** and **v** is denoted by $u \cdot v$ and is defined as

$$u \cdot v = |u| \cdot |v| \cdot \cos\theta$$

or in terms of coordinates as

$$u \cdot v = u_x v_x + u_y v_y + u_z v_z ,$$

where $u = (u_x; u_y; u_z)$, $v = (v_x; v_y; v_z)$.

Besides,

$u \cdot v = 0 \Leftrightarrow u$ and v are perpendicular.

Question

1 Find the angle between A=2i+2j−k and B=6i−3j+2k.

Solution

$\cos\theta = \dfrac{A \cdot B}{|A||B|}$;

A=2i+2j−k =(2; 2;-1);

B=6i−3j+2k= (6; -3; 2);

A·B= 2*6+2*(-3)+(-1)*2=12-6-2=4;

|A|= $\sqrt{2^2 + 2^2 + (-1)^2} = \sqrt{9} = 3$;

|B|= $\sqrt{6^2 + (-3)^2 + 2^2} = \sqrt{49} = 7$;

$\cos\theta = \dfrac{4}{3*7} = \dfrac{4}{21}$;

$\theta = \arccos\dfrac{4}{21}$, hence $\theta \approx 79.02^\circ$.

Answer: $\theta = \arccos\dfrac{4}{21}$, $\theta \approx 79.02^\circ$.

Question

2 Determine the value of a so that A=2i+aj+k and B=4i−2j−2k are perpendicular

Solution

A=2i+aj+k=(2; a;1)

B=4i−2j−2k=(4; -2; -2)

A·B =2*4+a*(-2)+1*(-2)=8-2a-2= 6-2a

A·B =0, then 6-2a=0; 2a=6; a=3.

Answer: a=3.

Question

3 Determine a unit vector perpendicular to the plane of A=2i−6j−3k and B=4i+3j−k

Solution

A=2i−6j−3k =(2; -6; -3)
B=4i+3j−k=(4; 3; -1)
Let u be a unit vector perpendicular to the plane of A=2i−6j−3k and B=4i+3j−k, then

$$u = \frac{A\times B}{|A\times B|};$$

$$A\times B = \begin{vmatrix} i & j & k \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix} = i\begin{vmatrix} -6 & -3 \\ 3 & -1 \end{vmatrix} - j\begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} + k\begin{vmatrix} 2 & -6 \\ 4 & 3 \end{vmatrix} =$$

$$= i(6+9) - j(-2+12) + k(6+24) = 15i - 10j + 30k$$

$$|A\times B| = \sqrt{15^2 + (-10)^2 + 30^2} = \sqrt{225+100+900} = \sqrt{1225} = 35;$$

$$u = \frac{A\times B}{|A\times B|} = \frac{1}{35}(15i - 10j + 22k) = \frac{15}{35}\text{i-}\frac{10}{35}\text{j+}\frac{30}{35}\text{k=}\frac{3}{7}\text{i-}\frac{2}{7}\text{j+}\frac{6}{7}\text{k;}$$

Answer:

u=$\frac{3}{7}$i-$\frac{2}{7}$j+$\frac{6}{7}$k.

Question

4 Find the work done in moving an object along a vector r=3i+2j−5k.

Solution

The object is moved by the force, and
the work down in moving an object along a vector r, if the applied force F, is defined by the dot product

A= F·r

Let the vector of applied force be F =xi+yj+zk=(x,y,z),

then A= F·r= 3x+2y-5z.

Answer: A= F·r= 3x+2y-5z for F =xi+yj+zk.

Question

5 Given that A=2i−j+3k and B=3i+2j−k, find A·B

Solution

A= 2i−j+3k=(2; -1; 3)
B=3i+2j−k=(3; 2; -1)

A·B =2*3+(-1)*2+3*(-1)=6-2-3=1;

Answer: A·B=1.