Answer on Question #59201 – Math – Analytic Geometry

Question

2. Given that $A_1 = 3i - 2j + k, A_2 = 2i - 4j - 3k, A_3 = -i + 2j + 2k$, find the magnitude of $2A_1 - 3A_2 - 5A_3$.

Solution

$2A_1 - 3A_2 - 5A_3 = 2(3i - 2j + k) - 3(2i - 4j - 3k) - 5(-i + 2j + 2k) = 6i - 4j + 2k - -6i + 12j + 9k + 5i - 10j - 10k = 5i - 2j + k.$

The magnitude of $2A_1 - 3A_2 - 5A_3$ is equal to $\sqrt{5^2 + (-2)^2 + 1^2} = \sqrt{30}$.

Answer: $\sqrt{30}$.

Question

3. Given that $A_1 = 2i - j + k, A_2 = i + 3j - 2k, A_3 = 3i + 2j + 5k,$ and $A_4 = 3i + 2j + 5k$, find scalars a, b, c such that $A_4 = aA_1 + bA_2 + cA_3$.

Solution

First of all, we shall check that A_1, A_2 and A_3 are linearly independent. We rewrite them in the coordinate form: $A_1 = (2, -1, 1), A_2 = (1, 3, -2), A_3 = (3, 2, 5)$. Then we calculate the determinant which consists of their coordinates:

$\Delta = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ 3 & 2 & 5 \end{vmatrix} = 30 + 6 + 2 - 9 + 8 + 5 = 42 \neq 0$. Since $\Delta \neq 0$ the vector A_4 can be represented as a linear combination of the vectors A_1, A_2 and A_3 in one way only. Since $A_4 = A_3$ we conclude that $A_4 = 0 \cdot A_1 + 0 \cdot A_2 + 1 \cdot A_3 \Leftrightarrow \begin{cases} a = 0 \\ b = 0. \\ c = 1 \end{cases}$

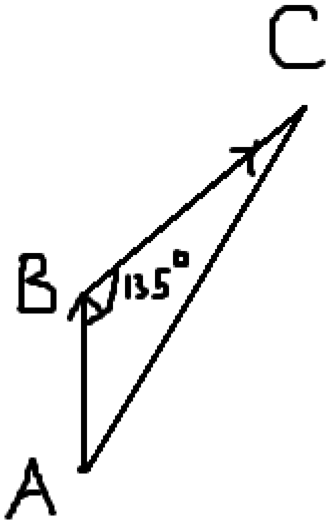
Answer: $a = 0, b = 0, c = 1$.

Question

5. A car travels 3 km due north, then 5 km northeast. Determine the resultant displacement.

Solution

The movement of a car is shown in the figure:



where $AB = 3, BC = 5, \angle ABC = 135^o$. The resultant displacement is AC.

Now we use the Cosine Rule:

$$AC = \sqrt{AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos \angle ABC} = \sqrt{9 + 25 - 30 \cdot \left(-\frac{\sqrt{2}}{2}\right)} =$$

$$= \sqrt{34 + 15\sqrt{2}} \approx 7.43 \text{ km.}$$

Answer: $\sqrt{34 + 15\sqrt{2}}$ km.

Question

6. If $A_1 = 3i - j - 4k, A_2 = -2i + 4j - 3k, A_3 = i + 2j - k$, find $|3A_1 - 2A_2 + 4A_3|$.

Solution

$3A_1 - 2A_2 + 4A_3 = 3(3i - j - 4k) - 2(-2i + 4j - 3k) + 4(i + 2j - k) = 9i - 3j - 12k +$
$+4i - 8j + 6k + 4i + 8j - 4k = 17i - 3j - 10k$.

Then $|3A_1 - 2A_2 + 4A_3| = \sqrt{17^2 + (-3)^2 + (-10)^2} = \sqrt{398}$.

Answer: $\sqrt{398}$.