Answer on Question #59070 – Math – Differential Equations

Question

3. Derive the differential equation associated with the primitive
y=Ax^3+Bx^2 + Cx + D
where A, B, C and D are arbitrary constants.

(a) D^3y/dx^2= 0
(b) d^4y/dx^4 + d^3y/dx^3 = 0
(c) d^3y/dx^3 + d^2y/dx^2 = 0
(d) d^4y/dx^4 = 0

Solution

$\frac{d^4}{dx^4}(Ax^3 + Bx^2 + Cx + D) = 0$, so the differential equation is $\frac{d^4y}{dx^4} = 0$.

Answer: (d) d^4y/dx^4 = 0.

Question

5. Derive the differential equation for the area bounded by the arc of a curve, the x- axis, and the two ordinates, one fixed and one variable, is equal to trice the length of the arc between the ordinates
(I) y=2√4+ (dx/dy)^2
(II) Y =√1+(d^2y/dx^2)^2
(III) Y =2 √1+ (dy/dx)^2
(IV) y=3 √2+(dy/dx)^2

Solution

$Area:\ S = \int_a^x y(x)dx.\ \ Length\ of\ arc:\ L = \int_a^x \sqrt{1 + y'(x)^2}dx.$

So $\int_a^x y(x)dx = 3\int_a^x \sqrt{1 + y(x)^2}dx \ \rightarrow\ \frac{d}{dx}\int_a^x y(x)dx = 3\frac{d}{dx}\int_a^x \sqrt{1 + y'(x)^2}dx \rightarrow$

$$y = 3\sqrt{1 + y'^2}\,.$$

Answer: $y = 3\sqrt{1 + y'^2}$.

Question

6 Find the differential equation of all straight lines at a unit distance from the origin
(i) (x dy/dx − y)^2=1/2)^2
(II) (x dy/dx − y)^2 =1+ (dy/dx)^2

(III) (3x dy/dx − y)^2=3+(dy/dx)^2
(IV) (2x d^2y/dx^2 − y)^2=1+(dy/dx)^2

Solution

Equation of the line: $y = ax + b$;

Distance from the line to origin: $d = \frac{|b|}{\sqrt{a^2+1}} = 1 \ \rightarrow \ \ b = \pm\sqrt{a^2+1}$

So $y = ax \pm \sqrt{a^2+1} \rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(ax \pm \sqrt{a^2+1}\right) \rightarrow \ \ y' = a$

and $y = y'x \pm \sqrt{y'^2+1} \rightarrow \ \ (y - xy')^2 = y'^2 + 1$

Answer: (II) (x dy/dx − y)^2 =1+ (dy/dx)^2