ANSWER ON QUESTION #59047 – MATH – DIFFERENTIAL EQUATIONS

QUESTION 1

Solve $y(xy+1)dx + x(1+xy+x^2y^2)dy = 0$

- **a)** $2x^2y^2\ln|y| - 2xy - 1 = Cx^2y^2$
- **b)** $3x^2y^2\ln|y| - 2xy - 3 = Cx^2y^2$
- **c)** $2x^2y^2\ln|y| - xy = Cx^2y^2$
- **d)** $2x^3y^2\ln|y| - 2xy - 1 = Cx^2y^5$

SOLUTION

In the equation, we introduce a new unknown function $y(x) = \frac{z(x)}{x}$

$$\frac{dy}{dx} = \frac{1}{x}\frac{dz}{dx} - \frac{z}{x^2}$$

$$y(xy+1)dx + x(1+xy+x^2y^2)dy = 0 \Leftrightarrow$$

$$y(xy+1) + x(1+xy+x^2y^2)\frac{dy}{dx} = 0 \Leftrightarrow$$

$$\frac{z}{x}\left(x\frac{z}{x}+1\right) + x\left(1 + x\frac{z}{x} + x^2\frac{z^2}{x^2}\right)\left(\frac{1}{x}\frac{dz}{dx} - \frac{z}{x^2}\right) = 0 \Leftrightarrow$$

$$\frac{z}{x}(z+1) + \frac{dz}{dx}(1+z+z^2) - x\frac{z}{x^2}(1+z+z^2) = 0 \Leftrightarrow$$

$$\frac{z}{x}(1+z-1-z-z^2) = -\frac{dz}{dx}(1+z+z^2) \Leftrightarrow -\frac{z^3}{x} = -\frac{dz}{dx}(1+z+z^2) \Leftrightarrow$$

$$\frac{dx}{x} = dz\frac{1+z+z^2}{z^3} = dz\left(\frac{1}{z^3} + \frac{1}{z^2} + \frac{1}{z}\right)$$

$$\ln|x| + \mathrm{CONST} = -\frac{1}{2z^2} - \frac{1}{z} + \ln|z| \Leftrightarrow \mathrm{CONST} = -\frac{1}{2x^2y^2} - \frac{1}{xy} + \ln|xy| - \ln|x|$$

$$CONST * 2x^2y^2 = -1 - 2xy + 2x^2y^2\ln|y| \Leftrightarrow 2x^2y^2\ln|y| - 2xy - 1 = \underbrace{2 * CONST}_{C} * x^2y^2$$

ANSWER:

a) $2x^2y^2\ln|y| - 2xy - 1 = Cx^2y^2$

QUESTION 2

Solve $xdy - ydx - \sqrt{x^2 - y^2}dx = 0$

a) $Cx = 2e^{arcsin\frac{y}{x}}$
b) $Cx = e^{arcsin\frac{y}{x}}$
c) $Cx = e^{arcsin\frac{2y}{3x}}$
d) $Cx = e^{arccos\frac{y}{x}}$

SOLUTION

In the equation, we introduce a new unknown function $y(x) = x * z(x)$

$$\frac{dy}{dx} = z + x\frac{dz}{dx}$$

$$xdy - ydx - \sqrt{x^2 - y^2}dx = 0 \Leftrightarrow x\frac{dy}{dx} - y - \sqrt{x^2 - y^2} = 0$$

$$x\left(z + x\frac{dz}{dx}\right) - zx - \sqrt{x^2 - x^2z^2} = 0 \Leftrightarrow xz + x^2\frac{dz}{dx} - zx - x\sqrt{1 - z^2} = 0$$

$$x^2\frac{dz}{dx} = x\sqrt{1 - z^2} \Leftrightarrow \frac{dz}{\sqrt{1 - z^2}} = \frac{dx}{x} \Leftrightarrow \arcsin\big(z(x)\big) = \ln|x| + \ln|C|$$

$$\ln|Cx| = arcsin\left(\frac{y}{x}\right) \Leftrightarrow Cx = e^{arcsin\left(\frac{y}{x}\right)}$$

ANSWER:

b) $Cx = e^{arcsin\left(\frac{y}{x}\right)}$

QUESTION 3

The population of student P at NOUN increases at a rate proportional to the population and to the addition of 150,250 and the population divided by 3, the differential equation of this statement is

a) $\frac{dP}{dT} = 3kP\frac{(150{,}250+P)}{4}$
b) $\frac{dP}{dT} = 2kP\frac{(150{,}250+P)}{3}$
c) $5\frac{dP}{dT} = kP\frac{(150{,}250+P)}{3}$
d) $\frac{dP}{dT} = 3kP\frac{(150{,}250+P)}{4}$

SOLUTION

The population of student P at NOUN increases at the population divided by 3. Its means that

$$\frac{dP}{dT} \sim \frac{P}{3}$$

The population of student P at NOUN increases at a rate proportional to the population and to the addition of 150,250 – its mean that

$$\frac{dP}{dT} \sim P + 150{,}250$$

THEN,

$$\frac{dP}{dT} \sim \frac{P}{3}(P + 150{,}250) \Leftrightarrow \frac{dP}{dT} = \underbrace{2k}_{the\ proportionality\ factor}\frac{P}{3}(P + 150{,}250).$$

ANSWER:

b) $\frac{dP}{dT} = 2kP\frac{(150{,}250+P)}{3}$