Answer on Question #59035 – Math – Differential Equations

Question

The differential equation for the curve given by the segment joining point Q(x,y) and the point of intersection of the normal at Q with the x –axis is bisected by the y –axis is given by

(a) 2y+x dx/dy=1/2y

(b) y+x dx/dy =1/2y

(c) y+x dx dy=1 3y

(d) y+3x dx/dy=1/2y

Solution

The equation of the normal at point (x_0, y_0) :

$$y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$$

We have:

$$y = 0 \quad \Rightarrow \quad x = -x_0$$

Then:

$$-y_0 = -\frac{1}{f'(x_0)}(-2x_0) \quad or$$

$$-y = \frac{2x}{y'} \qquad \Rightarrow y' = \frac{dy}{dx} = -\frac{2x}{y}$$

$$\frac{dx}{dy} = -\frac{y}{2x} \qquad \Rightarrow \qquad x\frac{dx}{dy} = -\frac{y}{2} \quad \Rightarrow \quad y + x\frac{dx}{dy} = \frac{y}{2}.$$

Answer: (b) $y + \ x\frac{dx}{dy} = \frac{1}{2}y$.

Question

For a particular element the rate of change of temperature(T) with respect to volume (V) is proportional to the vapor temperature and inversely proportional to the square of the volume, this phenomenon as a differential equation is

(a) dT/dV=kT/V^2

(b) dT/dV=kT/2V^2

(c) 2 dT/dV=kT/V^3

(d) dT/dV=3 kT/V^2

Solution

Differential equation is

$\frac{dT}{dV} = \frac{kT}{V^2}$,

where $k \neq 0$;

$\frac{dT}{kT} = \frac{dV}{V^2}$;

$\int \frac{dT}{kT} = \int \frac{dV}{V^2}$;

$\frac{1}{k} \ln T = -\frac{1}{V} + C$,

where C is an integration constant;

$\ln \sqrt[k]{T} = -\frac{1}{V} + C$,

$\sqrt[k]{T} = e^{-\frac{1}{V}+C}$,

$T = e^{-\frac{k}{V}+kC}$.

Solution of the differential equation is

$T = C_1 e^{-\frac{k}{V}}$,

where $C_1 = e^{kC}$ is a constant.

Answer: (a) $\frac{dT}{dV} = \frac{kT}{V^2}$.