Answer on Question #59023 – Math – Statistics and Probability

Question

7. In a study of plants, five characteristics are to be examined. If there are six recognizable differences in each of four characteristics and eight, recognizable difference in the remaining characteristics. How many plants can be distinguished by these five characteristics?

Solution

The number of plants can be distinguished by these five characteristics is

$N = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 8 = 10368.$

Answer: $10368.$

Question

8 Let X have a uniform distribution on the interval [A,B]. Compute V(X)

Solution

Because X is uniformly distributed in [A,B], then

$$f(x) = \frac{1}{B-A}, A \leq x \leq B; f(x) = 0, \text{ otherwise.}$$

$$\mu = \int_A^B \frac{xdx}{B-A} = \frac{1}{B-A}\left(\frac{x^2}{2}\right)_A^B = \frac{1}{2}\frac{1}{B-A}(B^2 - A^2) = \frac{A+B}{2}.$$

$$V(X) = \int_A^B \frac{(x-\mu)^2 dx}{B-A} = \frac{1}{B-A}\int_A^B (x-\mu)^2 d(x-\mu) = \frac{1}{3}\frac{1}{B-A}\left[\left(B - \frac{A+B}{2}\right)^3 - \left(A - \frac{A+B}{2}\right)^3\right] = \frac{(B-A)^2}{12}.$$

Answer: $\frac{(B-A)^2}{12}$.

Question

9 Let X have a standard gamma distribution with α=7. Compute P(X<4 or X>6)

Solution

The distribution function F of the standard gamma distribution with shape parameter α is given by

$$P(X < 4) = F(4);\ P(X > 6) = 1 - P(X < 7) = 1 - F(6).$$

$$\mathrm{P(X < 4 \text{ or } X > 6)} = 1 + F(4) - F(6) = 1 + 0.1107 - 0.3937 = 0.7170.$$

We used Excel function GAMMA.DIST to calculate the values of the cumulative distribution function:

$F(6)$=GAMMA.DIST(6,7,1,TRUE) and $F(4)$=GAMMA.DIST(4,7,1,TRUE).

Answer: $0.7170.$

Question

10 Let X = the time between two successive arrivals at the drive –up window of a bank. If X has a exponential distribution with h=1 (which is identical to a standard gamma distribution with a=1). Compute the standard deviation of the time between successive arrivals

Solution

The probability density function (pdf) of an exponential distribution is

$$f(x) = he^{-hx}, x \geq 0; 0, otherwise,$$

where h is mean.

In our case: $f(x) = e^{-x}$.

$$V(X) = \int_0^\infty (x-1)^2 e^{-x} dx = |y = (x-1)| = \int_{-1}^\infty y^2 e^{-y-1} dy = \frac{1}{e} \int_{-1}^\infty y^2 e^{-y} dy$$

$$\int_{-1}^\infty y^2 e^{-y} dy = -\int_{-1}^\infty y^2 de^{-y} = (-y^2 e^{-y})_{-1}^\infty + 2\int_{-1}^\infty ye^{-y} dy$$

$$(-y^2 e^{-y})_{-1}^\infty = e$$

$$\int_{-1}^\infty ye^{-y} dy = (-ye^{-y})_{-1}^\infty + \int_{-1}^\infty e^{-y} dy$$

$$(-ye^{-y})_{-1}^\infty = -e$$

$$\int_{-1}^\infty e^{-y} dy = (-e^{-y})_{-1}^\infty = e$$

$$V(X) = \tfrac{1}{e}[e + 2(e - e)] = 1.$$

The standard deviation is

$\sigma = \sqrt{V(X)} = \sqrt{1} = 1.$

Answer: 1.