Answer on Question #59022 – Math – Statistics and Probability

Question

5. A college lecturer never finishes his lecture before the end of the hour and always finishes his lecture within 2 min after the hour. Let X= the time that elapses between the end of the hour and the end of the lecture and suppose the pdf of X is

$$f(x) = \begin{cases} kx^2, & 0 \le x \le 2 \\ 0, & otherwise \end{cases}$$

what is the probability that the lecture continues for at least 90 sec beyond the end of the hour?

Solution

$$1 = \int_0^2 kx^2 dx = k\left(\frac{x^3}{3}\right)_0^2 = \frac{8}{3}k \to k = \frac{3}{8}$$

$$P(X \ge 1.5) = \frac{3}{8}\int_{1.5}^2 x^2 dx = \frac{3}{8}\left(\frac{x^3}{3}\right)_{1.5}^2 = \frac{2^3 - 1.5^3}{8} = 0.578125 = \frac{37}{64}.$$

Answer: $\frac{37}{64}$.

Question

6. Let X be a continuous rv with cdf

$$F(x) = \begin{cases} 0, x \le 0 \\ \frac{x}{4}\left[1 + \ln\left(\frac{4}{x}\right)\right], & 0 < x \le 4 \\ 1, x > 4 \end{cases}$$

What is the pdf of X?

Solution

$$f(x) = \frac{dF}{dx} = \begin{cases} \frac{1}{4}\left[1 + \ln\left(\frac{4}{x}\right)\right] + \frac{x}{4}\left[\frac{x}{4}\left(-\frac{4}{x^2}\right)\right] = \frac{1}{4}\ln\left(\frac{4}{x}\right), 0 < x \le 4, \\ 0, \ otherwise. \end{cases}$$

Answer: $f(x) = \frac{1}{4}\ln\left(\frac{4}{x}\right), 0 < x \le 4$, $f(x) = 0$, otherwise.