Answer on Question #59019 – Math – Statistics and Probability

Question

1. A certain shop repairs both audio and video components. Let A denote the event that the next component brought in for repair is an audio component, and let B be the event that the next component is a compact disc player (so the event B is contained in A). Suppose that P(A)= 0.6 and P(B) =0.05. What is P(B|A)?

Solution

$P(B|A) = \frac{\mathrm{P(B\cap A)}}{\mathrm{P(A)}} = \frac{\mathrm{P(B)}}{\mathrm{P(A)}} = \frac{0.05}{0.6} = \frac{1}{12} \approx 0.0833.$

Answer: $\frac{1}{12} \approx 0.0833.$

Question

2. Components of a certain type are shipped to a supplier in batches of ten. Suppose that 50% of all such batches contain no defective components. 30% contain one defective component, and 20% contain two defective components. Two components from a batch are randomly selected and tested. What are the probabilities associated with 0, 1, and 2 defective components being in the batch under the condition that neither tested component is defective.

Solution

Let B_0 be the event that the batch has 0 defectives, B_1 be the event the batch has 1 defective, and B_2 be the event the batch has 2 defectives. Let D_0 be the event that neither selected component is defective.

$$P(B_0) = 0.5, P(B_1) = 0.3, P(B_2) = 0.2$$

The event D_0 can happen in three different ways: (i) Our batch of 10 is perfect, and we get no defectives in our sample of two; (ii) Our batch of 10 has 1 defective, but our sample of two misses them; (iii) Our batch has 2 defective, but our sample misses them.

For (i), the probability is $(0.5)(1)$.

For (ii), the probability that our batch has 1 defective is 0.3. Given that it has 1 defective, the probability that our sample misses it is $\frac{\binom{9}{2}}{\binom{10}{2}}$, which is $\frac{8}{10}$. So the probability of (ii) is $(0.3)\left(\frac{8}{10}\right)$.

For (iii), the probability our batch has 2 defective is 0.2. Given that it has 2 defective, the probability that our sample misses both is $\frac{\binom{8}{2}}{\binom{10}{2}}$, which is $\frac{56}{90}$. So the probability of (iii) is $(0.2)\left(\frac{56}{90}\right)$. We have therefore found that

$$P(D_0) = (0.5)(1) + (0.3)\left(\frac{8}{10}\right) + (0.2)\left(\frac{56}{90}\right).$$

We use the general conditional probability formula:

$$P(B_0|D_0) = \frac{P(B_0 \cap D_0)}{P(D_0)} = \frac{(0.5)(1)}{(0.5)(1) + (0.3)\left(\frac{8}{10}\right) + (0.2)\left(\frac{56}{90}\right)} = 0.5784.$$

$$P(B_1|D_0) = \frac{P(B_1 \cap D_0)}{P(D_0)} = \frac{(0.3)\left(\frac{8}{10}\right)}{(0.5)(1)+(0.3)\left(\frac{8}{10}\right)+(0.2)\left(\frac{56}{90}\right)} = 0.2776.$$

$$P(B_2|D_0) = \frac{P(B_2 \cap D_0)}{P(D_0)} = \frac{(0.2)\left(\frac{56}{90}\right)}{(0.5)(1)+(0.3)\left(\frac{8}{10}\right)+(0.2)\left(\frac{56}{90}\right)} = 0.1440.$$

Answer: $0.5784;\ 0.2776;\ 0.1440.$