Answer on Question #59015 - Math - Statistics and Probability

Question

We want to test the hypothesis that the mean weight of a product is 16g against the hypothesis that it is 15.7 g. A sample of 49 products gives us a mean weight of 15.85 with a standard of deviation of 0.7 g. Find the critical region of the test with a significance level of 10%. What is the type II error?

Solution

The null hypothesis is H_0 : m=15.7 , the alternative hypothesis is H_a : $m=16 \ (m>15.7)$ and we have 49-1=48 degrees of freedom.

T-critical is t(0.1,48) = 1.299.

The critical region is

$$\bar{x} > \mu_0 + t^* \frac{s}{\sqrt{n}} = 15.7 + 1.299 \frac{0.7}{\sqrt{49}} = 15.83$$
 $\bar{x} > 15.83$

In our case $\bar{x} = 15.85$, thus we reject the null hypothesis.

The type II error is the failure to reject a false null hypothesis.

$$\frac{c - \mu_0}{\frac{s}{\sqrt{n}}} = -1.299$$

$$c = \mu_0 - 1.299 \frac{s}{\sqrt{n}} = 15.7 - 1.299 \frac{0.7}{\sqrt{49}} = 15.57.$$

$$\beta = 1 - P\left(t > \frac{15.57 - 16}{\frac{0.7}{\sqrt{49}}}\right) = 1 - P(t > -0.43) = 1 - 0.6654 = 0.3346.$$

Answer: $(15.83; +\infty)$, 0.3346.