Answer on Question #59009 – Math – Statistics and Probability

Question

When a customer places an order with Candy's on-line supermarket, a computerized accounting information system automatically checks to see if the customer has exceeded his or her credit limit. Past records indicate that the probability of customers exceeding their credit limit is 0.05. Suppose that on a given day 20 customers places orders.

1. What are the mean and standard deviation of the numbers of customers exceeding their credit limits?

2. What is the probability that zero customers will exceed their limits?

3. What is the probability that one customer will exceed his or her limit?

4. What is the probability that at least two customers will exceed their limits?

Solution

In this problem we have the binomial distribution with the next parameters: $n = 20, \ p = 0.05, \ q = 1 - p = 0.95$. Let ξ be the number of customers exceeding their credit limits.

1. $E\xi = np = 20 \cdot 0.05 = 1; \ \sigma_\xi = \sqrt{npq} = \sqrt{20 \cdot 0.05 \cdot 0.95} \approx 0.97.$

2. Since $p = 0.05 < 0.1; np = 1 < 10$, we can use the approximation of Poisson. We have:

$P\{\xi = k\} \approx \frac{(np)^k}{k!} e^{-np} = \frac{1}{k!} e^{-1}$. Then $P\{\xi = 0\} \approx \frac{1}{0!} e^{-1} = \frac{1}{e} \approx 0.368.$

3. $P\{\xi = 1\} \approx \frac{1}{1!} e^{-1} = \frac{1}{e} \approx 0.368.$

4. $P\{\xi \geq 2\} = 1 - P\{\xi < 2\} = 1 - (P\{\xi = 0\} + P\{\xi = 1\}) = 1 - (0.368 + 0.368) = 0.264.$

Answer:

1. 1; 0.97.

2. 0.368.

3. 0.368.

4. 0.264.