Answer on Question #58983 – Math – Abstract Algebra

Question

Is H=<Q', *> is a subgroup of G=<R, +>?

Solution

If H=<Q', *> is a subgroup of G=<R, +>, then <Q', *> must be the group on the operation + specified in G=<R, +>.

1. Closure: $\forall g_1, g_2 \in \mathrm{H} =< \mathrm{Q}', * >,\ g_1 + g_2 \in \mathrm{H} =< \mathrm{Q}', * >$.

2. Identity element: H=<Q', *> contains 0.
   $\forall g_1, g_2 \in \mathrm{H} =< \mathrm{Q}', * >,\ g_1 + (-g_1) = 0 \in \mathrm{H} =< \mathrm{Q}', * >$.

3. Inverse element: $\forall g_1 \in \mathrm{H} =< \mathrm{Q}', * >,\ (g_1)^{-1} = (-g_1) \in \mathrm{H} =< \mathrm{Q}', * >$.

4. Associativity: $\forall g_1, g_2, g_3 \in \mathrm{H} =< \mathrm{Q}', * >,\ \mathrm{g}_1 + (g_2 + g_3) = (g_1 + g_2) + g_3$ holds.

Hence the four properties of the subgroup criteria all hold, so H=<Q', *> is a subgroup of

G=<R, +>.