Answer on Question #58979 – Math – Abstract Algebra

Question

Do the odd integers form a group w.r.t addition?

Solution

Since odd integer a, b can be written as follows:

$$a = 2n + 1, \quad b = 2m + 1, \quad n, m = 0, \pm 1, \pm 2, \dots$$

we obtain

$$a + b = 2(n + m + 1) \neq 2k + 1, \quad k = 0, \pm 1, \pm 2, \dots$$

Therefore, addition of two odd integers yields even integer. Closure axiom is not satisfied, therefore the odd integers do not form a group w.r.t addition.

Answer: the odd integers do not form a group w.r.t addition.

Question

Do the non-zero positive real numbers form a group w.r.t multiplication?

Solution

Given any non-zero positive real numbers: $a \in R^+, b \in R^+$.

We have to check group axioms w.r.t multiplication:

1. Closure:
   We have $a \cdot b = c$, where $c \in R^+$ is some non-zero positive real number.
   Axiom is satisfied.
2. Identity element:

   We have $a \cdot 1 = 1 \cdot a = a$. Thus, identity element is $e = 1$. Axiom is satisfied.

3. Inverse element:

   We have $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = e$. Thus inverse element is $a^{-1} = \frac{1}{a}$. Axiom is satisfied.

4. Associativity

   As $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ is obviously satisfied for any $a \in R^+, b \in R^+, c \in R^+$.

   Axiom is satisfied.

Therefore, non-zero positive real numbers satisfy all group axioms w.r.t multiplication.

The non-zero positive real numbers form a group w.r.t multiplication.

Answer: The non-zero positive real numbers form a group w.r.t multiplication.