Answer on Question #58949 – Math – Statistics and Probability

Question

1. If 55% of companies in Tokyo own an answering machine, what is the probability that 3 of the next 4 randomly selected companies in Tokyo will own an answering machine?

Solution

It is binomial distribution.

$$P(k=3)=\frac{4!}{3!\,(4-3)!}0.55^3(1-0.55)^{4-3}=0.299475$$

Answer: 0.299475.

Question

2. A red die and a green die are rolled once. Find the conditional probability that the number on red die is odd, given that the sum of the two numbers showing is 9

Solution

GREEN	RED
3	6
4	**5**
5	4
6	**3**

These are the only possible ways to get 9 in total - four equally likely ways to produce 9. The red die is odd in two cases (cases are bold).

Given that the sum of two numbers is 9, the conditional probability of 'the red die is odd' is $\frac{1}{2}$ because half of the four cases show an odd red:

$$P(\text{red die is odd}|\text{sum of two numbers is } 9)=\frac{2}{4}=\frac{1}{2}.$$

Answer: $\frac{1}{2}$.

Question

3. A bus starts with 8 people and stops at 9 different stops. How many different ways can the 8 people depart if no two passengers can leave at the same bus stop.

Solution

There exist 9 ways for a stop at which no passenger gets off the bus.

Now, in $8!$ ways the 8 passengers can be arranged (ordered) to get off at distinct stops (1 per stop).

So the overall answer is the product

$$9 \cdot 8! = 362880.$$

Answer: $9 \cdot 8! = 362880.$

Question

4. Let A and B be any two events defined on the same sample space. Suppose P(A) = 0.3 and P(A U B) = 0.6. Find P(B) such that A and B are independent.

Solution

$$\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = P(A)P(B),$$

because A and B are independent events.

Probability of A U B is

$$\mathrm{P}(\mathrm{A\ U\ B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A})\mathrm{P}(B) = \mathrm{P}(\mathrm{A}) + \big(1 - \mathrm{P}(A)\big)\mathrm{P}(\mathrm{B}),$$

hence

$$\mathrm{P}(\mathrm{B}) = \frac{\mathrm{P}(\mathrm{A\ U\ B}) - \mathrm{P}(\mathrm{A})}{1 - \mathrm{P}(\mathrm{A})} = \frac{0.6 - 0.3}{1 - 0.3} = \frac{3}{7}.$$

Answer: $\frac{3}{7}$.