Answer on Question #58943 – Math – Trigonometry

Question

Solve triangle ABC which has angle A = 250.251°, angle B = 600.511° and a = 3.82 linear units. Find c.

Solution

The given triangle is defined with two angles and one side. This problem has one solution if the sum of each two triangle's angles is less than 180°, otherwise the problem doesn't have a solution.

Angle A = 250.251° > 180°. A triangle with this inside angle does not exist. We can assume that the given angle A is a non-convex angle contained by two sides of the triangle (b and c). Therefore, the corresponding inside angle of the triangle is equal to

$$\angle A' = 360^\circ - \angle A = 360^\circ - 250.251^\circ = 109.749^\circ.$$

The value of angle B ($\angle B = 600.511^\circ > 360^\circ$) can be transformed as follows:

$$\angle B = 600.511^\circ = 600.511^\circ - 360^\circ = 240.511^\circ.$$

Since $\angle B = 240.511^\circ > 180^\circ$, a triangle with such an inside angle doesn't exist. Again, we can assume that angle B is a non-convex angle contained by the triangle's sides a and c, so that the corresponding inside angle is as follows:

$$\angle B' = 360^\circ - \angle B = 360^\circ - 240.511^\circ = 119.489^\circ.$$

The sum of known inside angles of the triangle:

$$\angle A' + \angle B' = 109.749^\circ + 119.489^\circ = 229.238^\circ > 180^\circ.$$

Therefore, the given triangle doesn't exist.

If we disregard the given values, the problem can be solved in the following way.

The unknown angle C is

$$\angle C = 180^\circ - \angle A - \angle B.$$

Given the length of side a and measures of angles A and C, the length of side c can be found from the theorem of sines:

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c};$$

hence

$$c = a\frac{\sin\angle C}{\sin\angle A} = a\frac{\sin(180^\circ - \angle A - \angle B)}{\sin\angle A}.$$

Answer: the given triangle doesn’t exist;

$c = a\frac{\sin(180^\circ - \angle A - \angle B)}{\sin\angle A}$.