

Answer on Question #58930 – Math – Trigonometry

Question

For the simple harmonic motion equation $d = 5\sin\left(\frac{\pi}{4}t\right)$, what is the frequency?

If necessary, use the slash (/) to denote a fraction. _____

Solution

$$d = 5\sin\left(\frac{\pi}{4}t\right) = a\sin(\omega t),$$

$$\omega = \frac{\pi}{4}.$$

$$\text{Frequency is } f = \frac{\omega}{2\pi} = \frac{\pi}{4} \cdot \frac{1}{2\pi} = \frac{1}{8} = 1/8 \text{ s}^{-1}.$$

Answer: $1/8 \text{ s}^{-1}$.

Question

Find a model for simple harmonic motion if the position at $t = 0$ is 0, the amplitude is 5 centimeters, and the period is 4 seconds.

$$d = 5\sin(4t)$$

$$d = 4\sin(5t)$$

$$d = 5\cos\left(\frac{\pi}{2}t\right)$$

$$d = 5\sin\left(\frac{\pi}{2}t\right)$$

Solution

The formula for simple harmonic motion is

$$d = A\sin(\omega t + \varphi),$$

where the amplitude is $A = 5$ centimeters, the period is $T = \frac{2\pi}{\omega} = 4$ seconds, hence $\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2} \text{ s}^{-1}$.

It is given that $d(0) = 0 \Rightarrow A\sin(\omega \cdot 0 + \varphi) = 0 \Rightarrow A\sin(\varphi) = 0$, hence $\varphi = 0$ or $\varphi = \pi$.

Thus, $d = 5\sin\left(\frac{\pi}{2}t\right)$.

Answer: $d = 5\sin\left(\frac{\pi}{2}t\right)$.

Question

Find a model for simple harmonic motion if the position at $t = 0$ is 6, the amplitude is 6 centimeters, and the period is 4 seconds.

$$d = 4\sin\left(\frac{\pi}{3}t\right)$$

$$d = 6\cos(4t)$$

$$d = 4\sin(6t)$$

$$d = 6\cos\left(\frac{\pi}{2}t\right)$$

Solution

The formula for simple harmonic motion is

$$d = A\sin(\omega t + \varphi),$$

where the amplitude is $A = 6$ centimeters, the period is $T = \frac{2\pi}{\omega} = 4$ seconds, hence $\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2} \text{ s}^{-1}$.

It is given that $d(0) = 6 \Rightarrow A\sin(\omega \cdot 0 + \varphi) = 6 \Rightarrow 6\sin(\varphi) = 6$, hence $\varphi = \frac{\pi}{2}$.

Thus, $d = 6\sin\left(\frac{\pi}{2}t + \frac{\pi}{2}\right) = 6\cos\left(\frac{\pi}{2}t\right)$.

Answer: $d = 6\cos\left(\frac{\pi}{2}t\right)$.