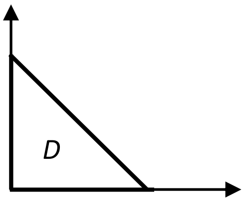
Answer on Question #:58903 – Math – Calculus

Question

Find f(x, y) such that z = f(x, y) defines a plane and ∬ f(x,y)dA =∬ x f(x,y) dA = 0, f(1,2) = −1, where D is the region bounded by the graphs of x + y = 3, x = 0, and y = 0.

Solution

If $f(x,y)$ is a plane, then $z = f(x,y) = ax + by + c$. Region D: $0 \le x \le 3,\ 0 \le y \le 3 - x$



Then $\iint_D f(x,y)dA = \int_0^3 dx \int_0^{3-x} (ax + by + c)dy = \int_0^3 dx \cdot \left(axy + \frac{1}{2}by^2 + cy\right)\Big|_0^{3-x} =$

$$= \int_0^3 \left(\frac{1}{2}b(x-3)^2 - c(x-3) - ax(x-3)\right) dx$$

$$= \left(\frac{1}{6}b(x-3)^3 - \frac{c}{2}(x-3)^2 - \frac{1}{3}ax^3 + \frac{3}{2}ax^2\right)\Big|_0^3 = \frac{3}{2}(a+b+c)$$

and $\iint_D xf(x,y)dA = \int_0^3 dx \int_0^{3-x} x(ax + by + c)dy = \int_0^3 dx \cdot \left(ax^2y + \frac{1}{2}bxy^2 + cxy\right)\Big|_0^{3-x} = \int_0^3 \left(\frac{1}{2}bx(x-3)^2 - cx(x-3) - ax^2(x-3)\right) dx = \int_0^3 \left(x\left(\frac{9b}{2} + 3c\right) - x^2(3b - 3a + c) - x^3\left(a - \frac{b}{2}\right)\right) = \left(\frac{x^2}{2}\left(\frac{9b}{2} + 3c\right) - \frac{x^3}{3}(3b - 3a + c) - \frac{x^4}{4}\left(a - \frac{b}{2}\right)\right)\Big|_0^3 = \frac{9}{8}(6a + 3b + 4c)$

and $f(1,2) = a + 2b + c$.

Then we have 3 conditions:

$$\frac{3}{2}(a+b+c) = 0$$

$$\frac{9}{8}(6a + 3b + 4c) = 0$$

$$a + 2b + c = -1$$

Solving the linear system above find $a = -\frac{1}{2},\ b = -1,\ c = \frac{3}{2}$.

Finally obtain $f(x,y) = ax + by + c = -\frac{1}{2}x - y + \frac{3}{2}$.

Answer: $f(x,y) = -\frac{1}{2}x - y + \frac{3}{2}$.