Answer on Question #58884 – Math – Differential Equations

Question

8. Solve

$y(xy+1)dx + x(1+xy+x^2y^2)dy = 0$

Solution

Transform the equation:

$$y' = -\frac{y(xy+1)}{x(1+xy+x^2y^2)}$$

Substitute:

$u(x) = xy \quad \Rightarrow \quad y = \frac{u}{x} \quad \Rightarrow \quad y' = \frac{u'x-u}{x^2}.$

Next,

$\frac{u'x-u}{x^2} + \frac{u(u+1)}{x^2(u^2+u+1)} = 0,$

$(u'x-u)(u^2+u+1) + u(u+1) = 0,$

$u'x(u^2+u+1) = u(u^2+u+1) - u(u+1),$

$u'x(u^2+u+1) = u^3,$

$\left(\frac{1}{u}+\frac{1}{u^2}+\frac{1}{u^3}\right) du = \frac{dx}{x},$

$\ln u - \frac{1}{u} - \frac{1}{2u^2} = \ln x + C.$

Substituting $u = xy$ obtain

$\ln(xy) - \frac{1}{xy} - \frac{1}{2x^2y^2} = \ln x + C.$

Answer: $\ln(xy) - \frac{1}{xy} - \frac{1}{2x^2y^2} = \ln x + C.$

Question

9. Solve

$xdy - ydx - \sqrt{x^2-y^2}dx = 0$

Solution

$xy' = y + \sqrt{x^2 - y^2},$

$y = tx \quad \Rightarrow \quad y' = t'x + t,$

$x(t'x + t) = tx + x\sqrt{1 - t^2},$

$t'x = \sqrt{1 - t^2},$

$\frac{dt}{\sqrt{1-t^2}} = \frac{dx}{x},$

$\sin^{-1} t = \ln|x| + \ln|C|.$

Using the previous equality and formula $t = \frac{y}{x}$ obtain

$\sin^{-1}\frac{y}{x} = \ln|Cx|.$

Answer:

$\sin^{-1}\frac{y}{x} = \ln|Cx|.$

Question

10. The population of student *p* at NUN increases at a rate proportional to the population and to the addition of 150,250 and the population divided by 3, the differential equation of this statement is.

Solution

The rate of increasing population:

$p' = \frac{dp}{dt} = k\left(p + 150250 + \frac{p}{3}\right).$

The differential equation is

$p' = k\left(\frac{4p}{3} + 150250\right).$

Answer:

$p' = k\left(\frac{4p}{3} + 150250\right).$