Answer on Question #58882 – Math – Differential Equations

Question

1. *H* grams of artificial sugar in water are being converted into dextrose at a rate which is proportional to the square of the amount unconverted. Find the differential equation expressing the rate of conversion after *v* minutes given that *s* grams is converted in *v* minutes and *c* being the constant of proportionality.

Solution

The Rate of conversion is given by the following differential equation

$$y' = c(H - y)^2.$$

Initial value:

$$y(v) = s.$$

Here *y* is the amount of converted sugar,

$y' = \frac{dy}{dt}$ is the rate of conversion.

Answer: $y' = c(H - y)^2, y(v) = s.$

Question

2. A vehicle of mass *m* moves along a straight line (the – axis) while subject to a force indirectly proportional to its displacement *x* from a fixed point O in its path and a resisting force proportional to its acceleration. Express the total force as a differential equation.

Solution

Total force:

$$F = ma = \frac{k_1}{x} - k_2 a$$

where

$a = x'' = \frac{d^2x}{dt^2}$ is the acceleration;

k_1, k_2 are the constants of proportionality.

Thus, the differential equation will be

$$mx'' = \frac{k_1}{x} - k_2 x''.$$

Answer: $mx'' = \frac{k_1}{x} - k_2 x''$.

Question

3. Derive the differential equation associated with the primitive

$y = Ax^3 + Bx^2 + Cx + D$

where *A, B, C* and *D* are arbitrary constants.

Solution

Given

$$y = Ax^3 + Bx^2 + Cx + D$$

compute

$y' = 3Ax^2 + 2Bx + C$,

$y'' = 6Ax + 2B$,

$y''' = 6A$,

where *A, B, C* are arbitrary constants.

From the last equality it follows that a differential equation is

$$\frac{d^4y}{dx^4} = 0.$$

Answer: $\frac{d^4y}{dx^4} = 0$.