Answer on Question #58827 – Math – Analytical Geometry

Question

5) If $\vec{A} = \left(51\hat{\imath} - 14\hat{\jmath} + 43\hat{k}\right)\,cm$ and $\vec{B} = \left(-31\hat{\imath} - 21\hat{\jmath} - 11\hat{k}\right)\,mm$.

a) Find $\vec{D} = -\vec{A} + 3\vec{B}$.

b) Find the magnitude $\vec{D}$.

Solution

a) From the condition of the question we can see that the components of vector $\vec{A}$ are given in centimeters and the components of vector $\vec{B}$ are given in millimeters. Let's first convert them into meters:

$$\vec{A} = \left(0.51\hat{\imath} - 0.14\hat{\jmath} + 0.43\hat{k}\right)\,m, \vec{B} = \left(-0.031\hat{\imath} - 0.021\hat{\jmath} - 0.011\hat{k}\right)\,m.$$

To find $\vec{D} = -\vec{A} + 3\vec{B}$, we first multiply vectors $\vec{A}$ and $\vec{B}$ by scalars -1 and 3 respectively. Then, we add these two vectors.

Let's multiply vectors $\vec{A}$ and $\vec{B}$ by scalars -1 and 3 respectively:

$$-1 \cdot \vec{A} = \left(-0.51\hat{\imath} + 0.14\hat{\jmath} - 0.43\hat{k}\right)\,m,$$

$$3 \cdot \vec{B} = \left(-0.093\hat{\imath} - 0.063\hat{\jmath} - 0.033\hat{k}\right)\,m.$$

Using formulas

$$\vec{A} = \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right),$$

$$\vec{B} = \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right),$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + \left(A_y + B_y\right)\hat{\jmath} + (A_z + B_z)\hat{k}$$

(when we add two vectors, we must add the components separately).

let's add vectors $-1 \cdot \vec{A}$ and $3 \cdot \vec{B}$:

$$\vec{D} = -\vec{A} + 3\vec{B} = \left(-0.51 + (-0.093)\right)\hat{\imath} + \left(0.14 + (-0.063)\right)\hat{\jmath} +$$

$$+\left(-0.43 + (-0.033)\right)\hat{k} = \left(-0.603\hat{\imath} + 0.077\hat{\jmath} - 0.463\hat{k}\right)\,m.$$

$\vec{D} = -\vec{A} + 3\vec{B} = \left(-0.603\hat{\imath} + 0.077\hat{\jmath} - 0.463\hat{k}\right) m.$

b) The magnitude of the vector $\vec{D}$ can be found using the Pythagorean theorem:

$$\left|\vec{D}\right| = \sqrt{D_x^2 + D_y^2 + D_z^2} = \sqrt{(-0.603\ m)^2 + (0.077\ m)^2 + (-0.463\ m)^2} = 0.764\ m.$$

Answer:

a) $\vec{D} = -\vec{A} + 3\vec{B} = \left(-0.603\hat{\imath} + 0.077\hat{\jmath} - 0.463\hat{k}\right) m.$

b) $\left|\vec{D}\right| = 0.764\ m.$