Answer on Question #58713 – Math – Statistics and Probability

Question

The mean time taken by all participants to run a road race was found to 220 minutes with a standard deviation of 20 minutes. Using Chebyshev's theorem, find the percentage of the runners who ran this road race in

a) 180 to 260 minutes

b) 160 to 280 minutes

c) 170 to 270 minutes.

Solution

For normal distributions, about 68% of results will fall between +1 and -1 standard deviations from the mean. About 95% will fall between +2 and -2 standard deviations. Chebyshev's Theorem allows us to use this idea for any distribution, even if that distribution isn't normal.

Chebyshev's Theorem tells us that no matter what the distribution looks like,

the probability that a randomly selected values is in the interval $\mu \pm k\sigma$ is at least $\left(1-\frac{1}{k^2}\right)$,

and the percentage is $\left[100\left(1-\frac{1}{k^2}\right)\right]$%.

We have $\mu = 220$ and $\sigma = 20$.

a) To find the percentage of the runners who ran this road race in 180 to 260 minutes, we need to find *k*.

We can see that $220-180=40$ and $260-220=40$. So the interval is symmetrical about the mean.

So the value of *k* is

$$k=\frac{220-180}{20}=\frac{40}{20}=2.$$

Chebyshev's Theorem now tells us that the percentage of the runners who ran this road race in 180 to 260 minutes is at least

$$\left[100\left(1-\frac{1}{k^2}\right)\right]=100\cdot\left(1-\frac{1}{2^2}\right)=100\cdot\left(1-\frac{1}{4}\right)=100\cdot(1-0.25)=100\cdot 0.75=75\%.$$

Answer: 75 %.

b) Let's find the percentage of the runners who ran this road race in 160 to 280 minutes.

We can see that $220-160=60$ and $280-220=60$. So the interval is symmetrical about the mean.

So the value of *k* is

$$k=\frac{220-160}{20}=\frac{60}{20}=3.$$

Chebyshev's Theorem now tells us that the percentage of the runners who ran this road race in 160 to 280 minutes is at least

$$\left[100\left(1-\frac{1}{k^2}\right)\right]=100\cdot\left(1-\frac{1}{3^2}\right)=100\cdot\left(1-\frac{1}{9}\right)=100\cdot(1-0.11)=100\cdot 0.89=89\%$$

Answer: 89 %.

c) Let's find the percentage of the runners who ran this road race in 170 to 270 minutes.

We can see that $220-170=50$ and $270-220=50$. So the interval is symmetrical about the mean.

So the value of *k* is

$$k=\frac{220-170}{20}=\frac{50}{20}=2.5.$$

Chebyshev's Theorem now tells us that the percentage of the runners who ran this road race in 170 to 270 minutes is at least

$$\left[100\left(1-\frac{1}{k^2}\right)\right]=100\cdot\left(1-\frac{1}{2.5^2}\right)=100\cdot\left(1-\frac{1}{6.25}\right)=100\cdot(1-0.16)=100\cdot 0.84=84\%.$$

Answer: 84 %.