Answer on Question #58664 – Math – Differential Equations

Question

Solve the intital value problem

(cosy)dy/dx-siny=x,y(0)=0

Solution

1) We have an equation

$$\cos y \cdot y' - \sin y = x,$$

that is,

$$(\sin y)' - \sin y = x.$$

Denote

$$u = \sin y.$$

We get a linear equation with respect to u:

$$u' - u = x. \qquad (1)$$

2) Multiplying the equation (1) by e^{-x} we have

$$e^{-x}u' - e^{-x}u = xe^{-x},$$

which is equivalent to

$$(e^{-x}u)' = xe^{-x}.$$

Integrating both parts with respect to *x*

$$e^{-x}u = \int xe^{-x}dx + C,$$

hence

$$e^{-x}u = -xe^{-x} - e^{-x} + C$$

and multiplying the equality by e^{x}

we get

$$u = -x - 1 + Ce^{x}.$$

3) Substituting $u = \sin x$ we have

$$\sin y = -x - 1 + Ce^x$$

and using initial conditions $x = 0, y = 0$ we get $C = 1.$

Finally obtain

$$\sin y = -x - 1 + e^x.$$

Answer: $\sin y = -x - 1 + e^x.$