Answer on Question #58626 – Math – Trigonometry

Question

Which function's graph has a period of 2?

$y = 2sin\pi x$
$y = 3cosx$
$y = -4sin2x$
$y = \cos(x - \frac{\pi}{2})$

Solution

Let the unknown sine function be $y = Asin(bx + c) + d$

Then the period: $T = \frac{t}{b}$, where t is a regular period of function (for example, regular period of sine and cosine t = 2π and for tangent and cotangent t = π).

Our job is to find a function whose period is equal to 2. So in our formula, T = 2. From the question it is clear that we need to look for these functions among the sine and cosine, then t=2π.

Substitute the values into the formula:

$2 = \frac{2\pi}{b}$,

hence

$b = \frac{2\pi}{2} = \pi$.

We also know that *b* is the coefficient of x, so we now find the function whose coefficient near x is π. That's only one function:

$y = 2sin\pi x$

Answer:

$y = 2sin\pi x$

Question

Which description matches the transformation $y = cosx$ undergoes to produce $y = 3\cos(-2x)$?

Reflection through the y-axis, vertical shift of 2 units, horizontal shift right by 3 units.

Horizontal shift left 2 units, then vertical shift up by 3 units.

Horizontal compression by factor $\frac{1}{2}$, vertical stretch by factor 3, then a reflection through the y-axis.

Horizontal stretch by factor 2, reflection through the x-axis, then vertical stretch by factor 3.

Solution

Transformations "after" the original function.

New function	How points in graph of $f(x)$ become points of new graph	visual effect
$f(x) + d$	$(a, b) \mapsto (a, b + d)$	shift up by d
$f(x) - d$	$(a, b) \mapsto (a, b - d)$	shift down by d
$cf(x)$	$(a, b) \mapsto (a, cb)$	stretch vertically by c
$\frac{1}{c}f(x)$	$(a, b) \mapsto (a, \frac{1}{c}b)$	shrink vertically by $\frac{1}{c}$
$-f(x)$	$(a, b) \mapsto (a, -b)$	flip over the x-axis

Transformations "before" the original function.

New function	How points in graph of $f(x)$ become points of new graph	visual effect
$f(x+d)$	$(a,b) \mapsto (a-d,b)$	shift left by d
$f(x-d)$	$(a,b) \mapsto (a+d,b)$	shift right by d
$f(cx)$	$(a,b) \mapsto (\frac{1}{c}a,b)$	shrink horizontally by $\frac{1}{c}$
$f(\frac{1}{c}x)$	$(a,b) \mapsto (ca,b)$	stretch horizontally by c
$f(-x)$	$(a,b) \mapsto (-a,b)$	flip over the y-axis

Suppose that we have a function $y = cosx$. To transform it, we do the following steps:

1. $y = \cos(2x)$: we have to shrink the function $y = cosx$ horizontally by $\frac{1}{2}$.
2. $y = 3\cos(2x)$: the function $y = \cos(2x)$ is stretched vertically by 3.
3. $y = 3\cos(-2x)$: the final step is to flip the function $y = 3\cos(2x)$ over the y-axis.

Answer:

Horizontal compression by factor $\frac{1}{2}$, vertical stretch by factor 3, then a reflection through the y-axis.