Answer on Question #58603 – Math – Abstract Algebra

Question

Prove that $H = \{a + ib \in \mathbb{C} | a^2 + b^2 = 1\}$ is a subgroup of $\mathbb{C}$, where $\mathbb{C}$ is the set of complex numbers, *H* is the set of numbers $a + bi$, where $a, b \in \varphi$. Show that *H* is a subgroup of non-zero real numbers under multiplication.

Solution

It is given that $H = \{a + ib \in \mathbb{C} | a^2 + b^2 = 1\}$. Let $G = \{c + id \in \mathbb{C} \mid c^2 + d^2 = 1\}$. Then $H * G$ will be

$$(a + ib) * (c + id) = ac + iad + icb - bd = ac - bd + (ad + cb)i$$

We will prove that $(ac - bd)^2 + (ad + cb)^2 = 1$:

$$\begin{aligned}(ac - bd)^2 + (ad + cb)^2 &= a^2c^2 + b^2d^2 - 2abcd + a^2d^2 + 2abcd + b^2c^2 \\ &= a^2(c^2 + d^2) + b^2(d^2 + c^2) = (a^2 + b^2)(c^2 + d^2);\end{aligned}$$

$(a^2 + b^2)(c^2 + d^2) = 1$, because $a^2 + b^2 = 1$ by the statement of the question and

$c^2 + d^2 = 1$ by the assumption.

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