Answer on Question #58576 – Math – Statistics and Probability

Question

During 2001, 61.3% OF US Households purchased ground coffee. These households spent an average of \$36.16 on ground coffee during the year ("Annual Product Preference Study". Progressive Grocer, May 1, 2002, 31). Consider the annual ground coffee expenditures for households purchasing ground coffee, assuming that these expenditures are approximately distributed as normal random variable with the mean of \$36.16 and a standard deviation of \$10.00.

a. Find the probability that a household spent less than \$25.00

b. Find the probability that a household spent more than \$50.00

c. What proportion of the household spent between \$30.00 and \$40.00

Solution

$$z_i = \frac{x_i - \mu}{\sigma};$$

a. $x < \$25.00$

$$z = \frac{25 - 36.16}{10} = -1.116$$

To find the probability that a household spent less than \$25.00, one needs to find *p*-value, which corresponds to the obtained *z*-score. This value can be obtained either from a standard normal table or via using technology, i.e. NORM.S.DIST function of MS Excel.

$p(x < \$25.00) = p(z < -1.116) = 0.1322$.

b. $x > \$50.00$

$$z = \frac{50 - 36.16}{10} = 1.384$$

To find the probability that a household spent more than \$50.00, first one needs to find *p*-value, which corresponds to the obtained *z*-score, and after that the obtained value should be subtracted from 1. The *p*-value can be obtained either from a standard normal table or via using technology, i.e. NORM.S.DIST function of MS Excel.

$p(x < \$50.00) = p(z < 1.384) = 0.9168$

$p(x > \$50.00) = 1 - p(x < \$50.00) = 1 - p(z < 1.384) = 1 - 0.9168 = 0.0832$

c. $$z_1 = \frac{30 - 36.16}{10} = -0.616;$$

$$z_2 = \frac{40 - 36.16}{10} = 0.384$$

To find the probability that a household spent between \$30.00 and \$40.00, one needs to find the *p*-values, which correspond to the obtained *z*-scores, and after that subtract them. The *p*-values can be obtained either from a standard normal table or via using technology, i.e. NORM.S.DIST function of MS Excel.

$p(x < \$30.00) = p(z < -0.616) = 0.2689$,

$p(x < \$40.00) = p(z < 0.384) = 0.6495$,

$p(\$30.00 < x < \$40.00) = p(x < \$40.00) - p(x < \$30.00) = p(z < 0.384) - p(z < -0.616) =$

$= 0.6495 - 0.2689 = 0.3806$.

Answer: a. 0.1322. **b.** 0.0832. **c.** 0.3806.

Question

Potholes requiring repair in a section of a national highway occur at an average rate of 3.2 potholes per kilometer.
a. What is the probability that there are no potholes that require repair in 5 km of the highway
b. What is the probability that at most three potholes require repair in 200m?

Solution

The probability of specific number of occurrences in a period can be determined via Poisson distribution:

$$P(k)=\frac{\lambda^k e^{-\lambda}}{k!}$$

a. The probability of zero potholes in 5 kilometers can be found using the Poisson distribution with $\lambda = 5\times\lambda_0 = 5\times 3.2 = 16$:

$$P(0\ in\ 5)=\frac{16^0 e^{-16}}{0!}=1.25\times 10^{-7}.$$

b. The probability of at most 3 holes in 0.2 km is equal to the sum of the probability values, which correspond to 0, 1, 2, and 3 holes in 0.2 km. These values can be found using the Poisson distribution with $\lambda = 0.2\times\lambda_0 = 0.2\times 3.2 = 0.64$:

$$P(0\ in\ 0.2)=\frac{0.64^0 e^{-0.64}}{0!}=0.5273;$$

$$P(1\ in\ 0.2)=\frac{0.64^1 e^{-0.64}}{1!}=0.3375;$$

$$P(2\ in\ 0.2)=\frac{0.64^2 e^{-0.64}}{2!}=0.1080;$$

$$P(3\ in\ 0.2)=\frac{0.64^3 e^{-0.64}}{3!}=0.0230;$$

The answer is
$P(\le 3) = 0.5273 + 0.3375 + 0.1080 + 0.0230 = 0.9958$.

Answer: a. $1.25 \cdot 10^{-7}$. **b.** 0.9958.