Answer on Question #58515 – Math – Algebra

Question

1. Find the nth term of a sequence whose first several terms are given 1/2, 3/4, 5/6, 7/8

Solution

$c_n = \frac{An}{Kn}$, where

$A_n = a_1 + 2(n-1)$; $a_1 = 1$;

$K_n = k_1 + 2(n-1)$; $k_1 = 2$;

Answer: $\frac{1+2(n-1)}{2+2(n-1)}$.

Question

2. Find the nth partial sum of the sequence given by An=1/n-1/(n+1)

Solution

$A_n = \frac{1}{n} - \frac{1}{n+1}$;

$A_1 = 1 - \frac{1}{2} = \frac{1}{2}$;

$S_2 = A_1 + A_2 = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} = 1 - \frac{1}{3}$;

$S_n = A_1 + A_2 + A_3 + \ldots + A_n = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{n-1} - \frac{1}{n} + \frac{1}{n} - \frac{1}{n+1} =$

$= 1 - \frac{1}{n+1} = \frac{n}{n+1}$

Answer: $\frac{n}{n+1}$.

Question

3. The 11th term of an arithmetic sequence is 52, and the 19th term is 92. Find the 1000th term

Solution

$a_n = a_1 + d\,(n - 1)$,

$a_{11} = a_1 + d(11 - 1) = a_1 + 10d$,

$a_{19} = a_1 + d(19 - 1) = a_1 + 18d$.

Since $a_{11} = 52$ and $a_{19} = 92$, we get the system of two equations:

$$\begin{cases} 52 = a_1 + 10d \\ 92 = a_1 + 18d \end{cases}$$

$$\begin{cases} a_1 = 52 - 10d \\ a_1 = 92 - 18d \end{cases}$$

$$52 - 10d = 92 - 18d$$

$$8d = 40$$

$$d = 5$$

$$a_1 = 52 - 10 \cdot 5 = 2$$

a_n=2+5(n-1)

a_{1000}=2+5(1000-1)=4997.

Answer: 4997.

Question

4. How many terms of the arithmetic sequence 5, 7, 9 must be added to get 572?

Solution

a_1 =5; d=2; a_n=572;

572=$\frac{n}{2}$ (a_1+ a_n);

572=$\frac{n}{2}$ (2·5+(n-1)·2);

572=5n+n(n-1);

0=n^2+4n-572;

0=(n-22)(n+26).

This gives n=22 or n=-26. However, n is the number of terms in this partial sum, then n=22.

Answer: 22.

Question

5. The third term of a geometric sequence is 63/4 and the sixth is 1701/32. Find the fifth term

Solution

A_3=$\frac{63}{4}$

A_6=$\frac{1701}{32}$

A_3= $a\,r^{3-1}$= $a\,r^2$

A_6= $a\,r^{6-1}$= $a\,r^5$

$$\begin{cases} \frac{63}{4} = ar^2 \\ \frac{1701}{32} = ar^5 \end{cases}$$

$$\frac{ar^5}{ar^2}=\frac{\frac{1701}{32}}{\frac{63}{4}}$$

$$r^3=\frac{27}{8}$$

r=3/2

63/4= a(3/2)^2

a=7

A_n=7(3/2)$^{n-1}$

$A_5=7(3/2)^{5-1}=7(3/2)^4=567/16.$

Answer: 567/16.

Question

6. Find the fraction that represents the rational number (2.351)

Solution

$$\frac{23}{10}+\frac{51}{1000}+\frac{51}{100000}+\frac{51}{10000000}+\frac{51}{1000000000}+\ldots$$

$A=\frac{51}{1000}$; $r=\frac{1}{100}$;

$$\frac{51}{1000}+\frac{51}{100000}+\frac{51}{10000000}+\frac{51}{1000000000}+\ldots=S=\frac{A}{1-r}=\frac{\frac{51}{1000}}{1-\frac{1}{100}}=\frac{\frac{51}{1000}}{\frac{99}{100}}=\frac{51}{1000}\cdot\frac{100}{99}=\frac{51}{990};$$

$$2.351=\frac{23}{10}+\frac{51}{1000}+\frac{51}{100000}+\frac{51}{10000000}+\frac{51}{1000000000}+\cdots=\frac{23}{10}+\frac{51}{990}=\frac{2328}{990}=\frac{388}{165}.$$

Answer: $\frac{388}{165}$.