Answer on Question #58514 – Math – Geometry

Question

1. The frustum of a right circular cone has a slant height of 9 ft. and the radii of the bases are 5 ft. and 7 ft.

a. Find the lateral area and total area of the frustum.

b. What is the altitude of this frustum?

c. Find the altitude of the cone that was removed to leave this frustum.

d. What is the volume of the entire cone?

Solution

a. The lateral area is

$$M = \pi\, s\,(R + r) = \pi\cdot 9\cdot (7 + 5) = 126\pi\, ft^2$$

The total area is

$$A = M + \pi R^2 + \pi r^2 = \pi(126 + 49 + 25) = 200\pi\ ft^2$$

b. The altitude of this frustum is

$$h = \sqrt{s^2 - (R - r)^2} = \sqrt{9^2 - (7 - 5)^2} = \sqrt{77}\, ft.$$

c. The altitude H of the cone that was removed to leave this frustum

$$\frac{h + h_r}{7} = \frac{h_r}{5}$$

$$5h + 5h_r = 7h_r$$

$$5h = 2h_r$$

$$h_r = \frac{5}{2}h = \frac{5\sqrt{77}}{2}$$

d. The volume of the entire cone is

$$V = \frac{1}{3}\pi h_{total}(R^2) = \frac{1}{3}\pi(h + h_r)(R^2) = \frac{1}{3}\pi\left(\sqrt{77} + \frac{5}{2}\sqrt{77}\right)(7^2) = \frac{343\pi}{6}\sqrt{77}\, ft^3.$$

Question

2. Consider the right pentagon pyramid. The sides of the upper and lower bases of the frustum are 4 and 10 inches, respectively, and the altitude of a lateral face is 6 inches. Find:

a. Lateral area of the frustum

b. Total area of the frustum

c. Volume of the frustum

d. Volume of the entire pyramid.

Solution

a. Lateral area of the frustum

$$A_L = \frac{1}{2} \cdot n(a+b)s = \frac{1}{2} \cdot 5(4+10)6 = 210\ in^2$$

b. Total area of the frustum

$$A = A_L + \frac{1}{4}na^2 \cot\frac{180}{n} + \frac{1}{4}nb^2 \cot\frac{180}{n} = 210 + \frac{1}{4}5(4^2+10^2)\cot\frac{180}{5} = 410\ in^2$$

c. Volume of the frustum

$$V = \frac{1}{3}h\left(A + A' + \sqrt{AA'}\right),$$

where

$$A = \frac{1}{4} \cdot 5 \cdot 4^2 \cdot \cot\frac{180}{5} = 28$$

$$A' = \frac{1}{4} \cdot 5 \cdot 10^2 \cdot \cot\frac{180}{5} = 172$$

$$h = \sqrt{6^2 - \left(\frac{10-4}{2}\right)^2} = \sqrt{27}$$

$$V = \frac{1}{3}\sqrt{27}\left(28 + 172 + \sqrt{28 \cdot 172}\right) = 424\ in^3$$

d. Volume of the entire pyramid.

$$V_{total} = \frac{1}{3}A'H$$

$$\frac{H}{10} = \frac{H-h}{4} \rightarrow H = \frac{5}{3}h = 5\sqrt{3}$$

$$V_{total} = \frac{1}{3}172 \cdot 5\sqrt{3} = 497\ in^2$$