Answer on Question #58513 – Math – Geometry

Question

1. A factory's pressure tank rests on the upper base of a vertical pipe whose inside diameter is 1 1/2 ft and whose length is 40 ft. The tank is a vertical cylinder surmounted by a cone, and it has a hemispherical base. If the altitudes of the cylinder and the cone are respectively 6 ft and 3 ft and all three parts of the tank have an inside diameter of 6 ft, find the volume of water in the tank and pipe when full.

Solution

Pipe is a cylinder, so its volume can be found as

$V_{cyl} = \frac{\pi d_{cyl}^2 L_{cyl}}{4}$,

where d = inner diameter of cylinder, L =its length (or altitude).

Tank is comprised of cylinder, cone and hemisphere. Volume of hemisphere

$V_h = \frac{\pi d_h^3}{12}$,

volume of cone $V_c = \frac{\pi d_c^2 L_c}{12}$, where d and L are same as for cylinder.

$$V_{tank} = V_{cyl} + V_c + V_h = \frac{\pi d_{cyl}^2 L_{cyl}}{4} + \frac{\pi d_c^2 L_c}{12} + \frac{\pi d_h^3}{12} = \frac{\pi}{12}\left(3d_{cyl}^2 L_{cyl} + d_c^2 L_c + d_h^3\right)$$

$$= as\ all\ parts\ of\ tank\ have\ same\ inner\ diameter\ d = \frac{\pi d^2}{12}\left(3L_{cyl} + L_c + d\right)$$

$$= \frac{\pi \cdot 6^2}{12}(3 \cdot 6 + 3 + 6) = 81\pi\ ft^3$$

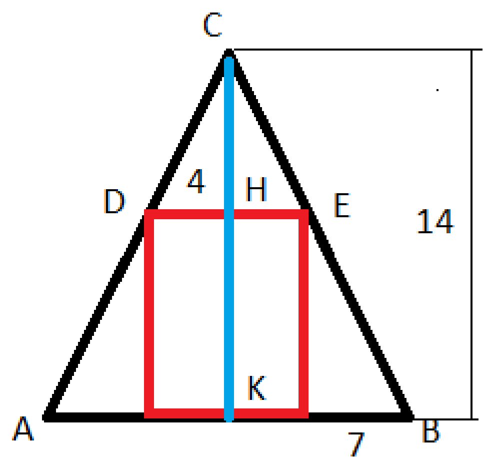
$$V_{pipe} = V_{cyl} = \frac{\pi d_{cyl}^2 L_{cyl}}{4} = \frac{\pi \cdot 1.5^2 \cdot 40}{4} = 22.5\pi\ ft^3$$

Answer $V_{tank} = 81\pi\ ft^3$; $V_{pipe} = 22.5\pi\ ft^3$; $V_{tot} = V_{tank} + V_{pipe} = 103.5\pi\ ft^3$

Question

2. A right circular cylinder whose diameter is 4 cm is cut from a right circular cone of height is 14 cm and radius 7 cm. What is the volume of the right circular cylinder?

Solution



On picture you can see the side look at cone and cylinder inside it. Blue line is altitude, red frame is side look at cylinder, black frame is side look at cone.

We can see that triangles CDE and CAB are similar. So height of triangle CDE is related to its base in the same ratio as the height of triangle CAB is related to its base:

$$\frac{CH}{DE} = \frac{CK}{AB}$$

We know that DE=4 cm (it is diameter of cylinder), CK=14 cm (altitude of cone), AB=14 cm is the diameter of cone's base. So,

$$CH = \frac{CK \cdot DE}{AB} = \frac{14 \cdot 4}{14} = 4\ cm$$

That means that altitude of cylinder is $HK = CK - CH = 14 - 4 = 10\ cm.$

Using formula for cylinder volume from the previous task obtain

$$V_{cyl} = \frac{\pi d_{cyl}^2 L_{cyl}}{4} = \frac{\pi \cdot 4^2 \cdot 10}{4} = 40\pi\ cm^3.$$