Answer on Question #58430 – Math – Differential Equations

Question

$y''' + y'' = 8x^2.$

Solution

The general solution will be the sum of the complementary solution and particular solution

$$y = y_{0+}y^*.$$

Find complementary solution $y''' + y'' = 0.$

Set up and solve the characteristic equation:

$k^3 + k^2 = 0,$

$k_1 = k_2 = 0,\ k_3 = -1,$

then

$y_0 = C_1 + C_2x + C_3e^{-x}.$

Find a particular solution of $y''' + y'' = 8x^2$ by the method of undetermined coefficients:

$y^* = x^2(Ax^2 + Bx + C) = Ax^4 + Bx^3 + Cx^2.$

Solve for unknown constants A, B, C:

$y^{*\prime} = 4Ax^3 + 3Bx^2 + 2Cx.$

$y^{*\prime\prime} = 12Ax^2 + 6Bx + 2C.$

$y^{*\prime\prime\prime} = 24Ax + 6B.$

Substitute the particular solution into the differential equation:

$24Ax + 6B + 12Ax^2 + 6Bx + 2C = 8x^2.$

$12Ax^2 + (24A + 6B)x + (6B + 2C) = 8x^2.$

Solve the system:

$$\begin{cases} 12A = 8, \\ 24A + 6B = 0, \\ 6B + 2C = 0. \end{cases} \Rightarrow \begin{cases} A = \dfrac{2}{3} \\ B = -4A = -\dfrac{8}{3} \\ C = -3B = 8 \end{cases}$$

Thus,

$$y^* = \frac{2}{3}x^4 - \frac{8}{3}x^3 + 8x^2,$$

then the general solution is given by

$$y = y_0 + y^* = C_1 + C_2 x + C_3 e^{-x} + \frac{2}{3}x^4 - \frac{8}{3}x^3 + 8x^2.$$

Answer: $y = C_1 + C_2 x + C_3 e^{-x} + \frac{2}{3}x^4 - \frac{8}{3}x^3 + 8x^2.$