Answer on Question #58422 – Math – Differential Equations

Question

6. Solve

$$(x^3 + y^3)dx - 3xy^2dy = 0$$

Solution

$(x^3 + y^3)dx = 3xy^2dy,\quad \frac{x^3+y^3}{3xy^2} = \frac{dy}{dx}\ ,\quad y' = \frac{1}{3}\left(\frac{x^2}{y^2} + \frac{y}{x}\right)$

Substitution $y = tx,\quad t = t(x),\quad y' = t + t'x,\quad t = \frac{y}{x}.$

We obtain $t + t'x = \frac{1}{3}\left(\frac{1}{t^2} + t\right),\quad t'x = \frac{1}{3}\left(\frac{1}{t^2} - 2t\right) = \frac{1-2t^3}{3t^2},\quad \frac{dt}{dx}x = \frac{1-2t^3}{3t^2},\quad \frac{3t^2dt}{(1-2t^3)} = \frac{dx}{x};$

$\int \frac{3t^2dt}{(1-2t^3)} = \int \frac{dx}{x},\quad -\frac{1}{2}\ln|1 - 2t^3| = \ln|Cx|,\quad 1 - 2t^3 = \frac{C}{x^2},\quad 1 - 2\frac{y^3}{x^3} = \frac{C}{x^2},$

$$x^3 - 2y^3 = Cx$$

Answer: $x^3 - 2y^3 = Cx.$

Question

7. Solve

$$(1 + 2e^{xy})dx + 2e^{xy}(1 - xy)dy = 0$$

5x+2yexy=C
x+2ye2xy=C
x+2yexy=C
5x+3yexy=C
(May be a bug)

Solution

According to the suggested answer options, this equation must be the total differential equation, or must become it through the introduction of some integrating factors. But we have:

$$d(5x + 2ye^{xy}) = (5 + 2y^2e^{xy})dx + 2e^{xy}(1 + xy)dy$$
$$d(x + 2ye^{2xy}) = (1 + 4y^2e^{2xy})dx + 2e^{2xy}(1 + 2xy)dy$$
$$d(x + 2ye^{xy}) = (1 + 2y^2e^{xy})dx + 2e^{xy}(1 + xy)dy$$
$$d(5x + 3ye^{xy}) = (5 + 3y^2e^{xy})dx + 3e^{xy}(1 + xy)dy$$

Answer: none of them.

Question

8. Solve

$$y(xy + 1)dx + x(1 + xy + x^2y^2)dy = 0$$

Solution

$$\frac{dy}{dx} = -\frac{y(xy + 1)}{x(1 + xy + x^2y^2)}$$

Substitution $xy = t,\quad t = t(x),\quad y = \frac{t}{x},\quad y' = \frac{t'x-t}{x^2} = \frac{t\prime}{x} - \frac{t}{x^2}.$

We obtain

$$\frac{t'}{x} - \frac{t}{x^2} = -\frac{t(t + 1)}{x^2(1 + t + t^2)};\ t'x - t = -\frac{t(t + 1)}{(1 + t + t^2)};\ t'x = t\left(1 - \frac{t + 1}{1 + t + t^2}\right);$$

$$\frac{dt}{dx}x = \frac{t^3}{1 + t + t^2};\quad \frac{(1 + t + t^2)dt}{t^3} = \frac{dx}{x};$$

$$\int \frac{(1+t+t^2)dt}{t^3} = \int \frac{dx}{x}, \quad -\frac{1}{2t^2} - \frac{1}{t} + \ln|t| = \ln|x| + C, \quad -\frac{1}{2t^2} - \frac{1}{t} + \ln\left|\frac{t}{x}\right| = C$$

$$2t^2 \ln\left|\frac{t}{x}\right| - 2t - 1 = t^2 C,$$

$$2x^2\, y^2 \ln|y| - 2xy - 1 = x^2 y^2 C$$

Answer: $2x^2\, y^2 \ln|y| - 2xy - 1 = Cx^2 y^2.$

Question

9. Solve

$$xdy - ydx - x^2 - y^2 - \sqrt{}dx = 0$$

$$Cx = 2e \arcsin yx$$

$$Cx = e \arcsin yx$$

$$Cx = e \arcsin^2 y^3 x$$

$$Cx = e \arccos yx$$

Answer: I think that the statement of this question is incorrect.

Question

10 The population of student P at NOUN increases at a rate proportional to the population and to the addition of 150,250 and the population divided by 3, the differential equation of this statement is

Solution.

$$\frac{dP}{dT} = kP(150{,}250 + P) : 3,$$

Answer: $\frac{dP}{dT} = kP(150{,}250 + P) : 3.$
