Answer on Question #58420 – Math – Differential Equations

Question

6. The solution to the differential equation $(4+t^2)\frac{dy}{dt}+2ty=4t$ is

y=4t24+t2+c6+t2

y=2t34+t3+c4+t2

y=2t27+t2

y=2t24+t2+c4+t2

Solution

$$(4+t^2)\frac{dy}{dt}+2ty=4t$$

$$(4+t^2)dy=2t(2-y)dt$$

$$\frac{dy}{2-y}=\frac{2t}{4+t^2}dt$$

$$\int\frac{dy}{2-y}=\int\frac{2t}{4+t^2}dt$$

$$-\ln(2-y)=\ln(t^2+4)+c_1$$

$$\frac{c_2}{2-y}=t^2+4$$

$$\frac{c_2}{t^2+4}=2-y$$

$$y=2-\frac{c_2}{t^2+4}=\frac{2t^2+8-c_2}{t^2+4}=\frac{2t^2}{t^2+4}+\frac{8-c_2}{t^2+4}$$

Answer: $y=\frac{2t^2}{t^2+4}+\frac{c}{t^2+4}$

Question

7. Find the general solution of differential equation $\frac{dy}{dx}-2y=4-x$

y=−12+23x+ce2x

y=−74+12x+ce2x

y=−74+22x+ce3x

y=−ln74+12x+celnx

Solution

$$\frac{dy}{dx} - 2y = 4 - x$$

$y = uv \ ; \quad y' = u'v + uv'$

$$u'v + uv' - 2uv = 4 - x$$

$$u'v + u(v' - 2v) = 4 - x$$

$$\begin{cases} v' - 2v = 0 \\ u'v = 4 - x \end{cases}$$

$$v' - 2v = 0 \Rightarrow \frac{dv}{dx} = 2v \quad \Rightarrow \quad \int \frac{dv}{2v} = \int dx \quad \Rightarrow \quad \frac{\ln v}{2} = x \quad \Rightarrow \quad v = e^{2x}$$

$$u' \cdot e^{2x} = 4 - x$$

$$u = \int \frac{4 - x}{e^{2x}} dx = \frac{2x - 7}{4e^{2x}} + c$$

$$y = uv = e^{2x}\left(\frac{2x - 7}{4e^{2x}} + c\right)$$

Answer: $y = ce^{2x} + \frac{x}{2} - \frac{7}{4}$.

Question

8. Solve the initial value problem $x\frac{dy}{dx} + 2y = 4x^2, \ \ y(1) = 2.$

y=x2+1x2,x>0
y=3x2+1x2,x>0
y=x−2+1x2,x>0
y=x3+13x2,x>0

Solution

$$x\frac{dy}{dx} + 2y = 4x^2$$

$$y' + \frac{2}{x}y = 4x$$

$y = uv \ ; \quad y' = u'v + uv'$

$$u'v + uv' + \frac{2uv}{x} = 4x$$

$$u'v + u\left(v' + \frac{2v}{x}\right) = 4x$$

$$\begin{cases} v' + \dfrac{2v}{x} = 0 \\ u'v = 4x \end{cases}$$

$$\frac{dv}{dx} = -\frac{2v}{x} \quad \Rightarrow \quad \int \frac{dv}{2v} = -\int \frac{dx}{x} \quad \Rightarrow \quad \frac{\ln v}{2} = -\ln x \quad \Rightarrow \quad v = \frac{1}{x^2}$$

$$u' \cdot \frac{1}{x^2} = 4x$$

$$u = \int 4x^3 dx = x^4 + c$$

$$y = uv = (x^4 + c) \cdot \frac{1}{x^2} = x^2 + \frac{c}{x^2}$$

$$y(1) = 1 + c = 2 \quad \Rightarrow \quad c = 1$$

Answer: $y = x^2 + \frac{1}{x^2}$, $x > 0$

Question

9. Solve for $\frac{dy}{dx} + y = 5sin2x$.

y=ce2x−sin2x−2cos3x
y=ce−x+tan2x−2cos2x
y=ce−x+sin2x+3cos2x
y=ce−x+sin2x−2cos2x

Solution

$$\frac{dy}{dx} + y = 5sin2x$$

$y = uv \ ; \quad y' = u'v + uv'$

$$uv + u'v + uv' = 5sin2x$$

$u'v + u(v + v') = 5sin2x$

$\begin{cases} v + v' = 0 \\ u'v = 5sin2x \end{cases}$

$v + v' = 0 \Rightarrow \frac{dv}{dx} = -v \quad \Rightarrow \quad \int \frac{dv}{v} = -x \quad \Rightarrow \quad \ln v = -x \quad \Rightarrow \quad v = e^{-x}$

$\frac{du}{dx} e^{-x} = 5sin2x$

$u = \int 5e^x \sin(2x)\, dx = e^x(sin2x - 2cos2x) + c$

$y = uv = e^{-x}(e^x(sin2x - 2cos2x) + c)$

Answer: $y = ce^{-x} + sin2x - 2cos2x.$

Question

10. Given the differential equation $\frac{dy}{dx} + \frac{1}{x}y = 3cos2x \quad , \quad x > 0$

the solution is

y=cx+3cos2x4x+3sin22

y=cx+3cos2x4x

y=cx+3sin22

y=c2x+3cosx4x+3sin62

Solution

$\frac{dy}{dx} + \frac{1}{x}y = 3cos2x \quad , \quad x > 0$

$y = uv \;\; ; \quad y' = u'v + uv'$

$\frac{1}{x}uv + u'v + uv' = 3cos2x$

$u'v + u\left(\frac{1}{x}v + v'\right) = 3cos2x$

$\begin{cases} \frac{1}{x}v + v' = 0 \\ u'v = 3cos2x \end{cases}$

$$\frac{dv}{dx} = -\frac{v}{x} \quad \Rightarrow \quad \int \frac{dv}{v} = -\int \frac{dx}{x} \quad \Rightarrow \quad \ln v = -\ln x \quad \Rightarrow \quad v = \frac{1}{x}$$

$$\frac{du}{dx} \cdot \frac{1}{x} = 3cos2x$$

$$u = \int 3x \cos(2x)\, dx = \frac{3}{4}(2xsin2x + cos2x) + c$$

$$y = uv = \frac{1}{x}\left(\frac{3}{4}(2xsin2x + cos2x) + c\right)$$

Answer: $y = \frac{c}{x} + \frac{3}{2}sin2x + \frac{3cos2x}{4x}$.