Answer on Question #58419 – Math – Differential Equations

Question

1. The differential equation

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dt}\right)^3 = x^2$$

is of order:
- 3
- 2
- 1
- 0

Solution

The order of a differential equation is the number of the highest derivative in a differential equation. Since the highest derivative is $\frac{d^2y}{dx^2}$ this equation is of the second order. So, the answer is 2.

Answer: 2.

Question

2. Any solution, which is obtained from the general solution by giving particular values to the arbitrary constants, is called?
- singular solution
- definite solution
- indefinite solution
- Particular solution

Solution

Any solution, obtained from the general solution by giving particular values to one or more of arbitrary constants, is called a particular solution.

Answer: particular solution.

Question

3. If in an ordinary or partial differential equation, the dependent variables and its derivatives occur to degree one only, and not as higher powers or products, the equation is said to be
- linear
- singular
- singleton
- non linear

Solution

When, in an ordinary or partial differential equation, the dependent variables and its derivatives occur to the degree one only, and not as higher powers or products, the equation is said to be linear.

Answer: linear.

Question

4. The _____ of a differential equation is the highest exponent of the highest order derivative appearing in it after the equation has been expressed in the form free from radicals and any fractional power of the derivatives or negative power.
- order
- total
- power
- degree

Solution

The degree of a differential equation is the highest exponent of the highest order derivative appearing in it after the equation has been expressed in the form free from radicals and any fractional power of the derivatives or negative power.

Answer: degree.

Question

5. Which of the following represent the solution of the differential equation?

$$\frac{d^2y}{dx^2} + 4y = 0.$$

- $5 \tan 2x + 5 \cos 2x$
- $5 \sin 2x + 4 \cos 2x$
- $5 \sin 2x - 3 \cos 2x$
- $5 \sin 22x - \cos 2x$

Solution

$$\frac{d^2y}{dx^2} + 4y = 0;$$
$$y'' + 4y = 0;$$

The characteristic equation is

$$k^2 + 4 = 0;$$
$$k^2 = -4;$$
$$k = \pm\sqrt{-4} = \pm 2i.$$

If $k = a + bi$ then $a = 0,\ b = 1.$

As solutions of the characteristic equation are complex numbers, the solution of the given homogeneous differential equation can be found as:

$$y_h = c_1 e^{ax} \cos bx + c_2 e^{ax} \sin bx = c_1 e^{0\cdot x} \cos 2x + c_2 e^{0\cdot x} \sin 2x = c_1 \cos 2x + c_2 \sin 2x,$$

where c_1, c_2 are arbitrary constants. The solution of the differential equation don't depend on c_1, c_2:

$$y_h = c_1 \cos 2x + c_2 \sin 2x,$$
$$y'_h = -2c_1 \sin 2x + 2c_2 \cos 2x,$$
$$y''_h = -4c_1 \cos 2x - 4c_2 \sin 2x,$$

Therefore,

$$y''_h + 4y_h = 0;$$
$$-4c_1 \cos 2x - 4c_2 \sin 2x + 4c_1 \cos 2x + 4c_2 \sin 2x = 0;$$
$$0 = 0.$$

So, the solution can be $y_h = 5 \sin 2x + 4 \cos 2x$ or $y_h = 5 \sin 2x - 3 \cos 2x$.

Answer: $y_h = 5 \sin 2x + 4 \cos 2x$ or $y_h = 5 \sin 2x - 3 \cos 2x$.