Answer on Question #58353 – Math – Analytic Geometry

The sides of a triangle lie on the lines $3x+4y+8$, $3x-4y-32=0$ and $x=8$. Find the equation of the circle inscribe in the triangle.

Solution

The triangle lying on the given lines is shown on Figure 1.

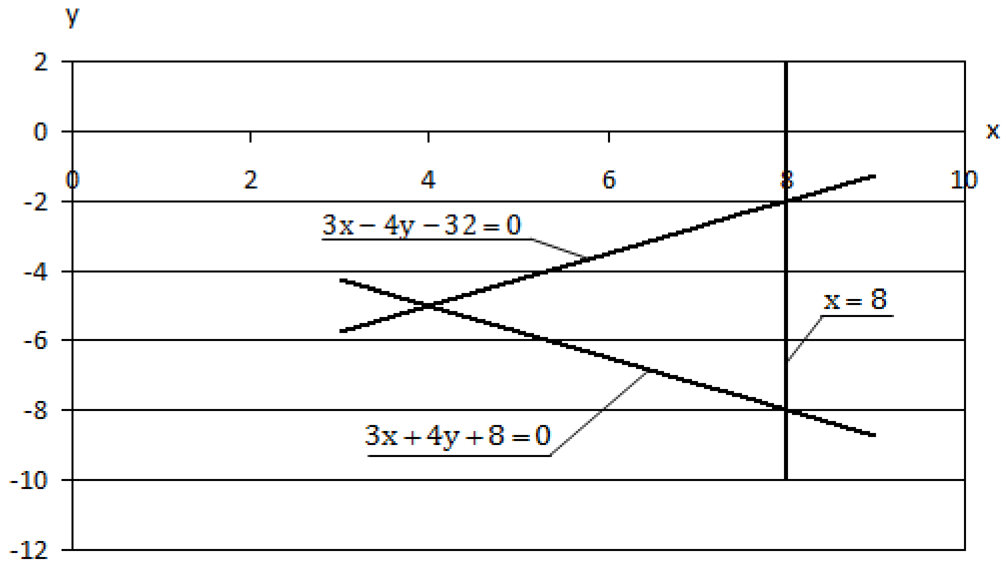

Figure 1. The triangle lying on the lines $3x+4y+8$, $3x-4y-32=0$ and $x=8$

The center of the inscribed circle is coincident with the intersection point of the bisectors of the triangle angles.

The equations of two possible bisectors of two angles formed by lines $A_1x+B_1y+C_1=0$ and $A_2x+B_2y+C_2=0$ can be found from the formula:

$$\frac{A_1x+B_1y+C_1}{\sqrt{A_1^2+B_1^2}}=\pm\frac{A_2x+B_2y+C_2}{\sqrt{A_2^2+B_2^2}}.$$

The equations for bisectors of two angles formed by lines $3x+4y+8=0$ and $3x-4y-32=0$ can be found as follows:

$$\frac{3x+4y+8}{\sqrt{3^2+4^2}}=\pm\frac{3x-4y-32}{\sqrt{3^2+4^2}};$$

$$3x+4y+8=\pm(3x-4y-32);$$

$$6x=24 \text{ and } 8y=-40;$$

$$x = 4 \text{ and } y = -5.$$

The bisecting line $x = 4$ lies beyond the area of the given triangle. Therefore, $y = -5$ is the equation of a bisector of the given triangle.

The equations for bisectors of two angles formed by lines $3x + 4y + 8 = 0$ and $x = 8$ can be found as follows:

$$\frac{3x + 4y + 8}{\sqrt{3^2 + 4^2}} = \pm \frac{x - 8}{\sqrt{1^2 + 0^2}};$$

$$\frac{3x + 4y + 8}{5} = \pm \frac{x - 8}{1};$$

$$3x + 4y + 8 = \pm(5x - 40).$$

Substituting the previously obtained equation of a bisector $y = -5$ in the latter equation, we will get the intersection point of the bisectors, which is the center of the inscribed circle:

$$3x + 4 \cdot (-5) + 8 = \pm(5x - 40);$$

$$2x = 28 \text{ and } 8x = 52;$$

$$x = 14 \text{ and } x = 6{,}5.$$

The point with coordinate $x = 14$ lies beyond the area of the given triangle. Therefore, the required center of the inscribed triangle has the coordinates: $x_C = 6{,}5;\ y_C = -5$.

The radius of the required circle is the distance between the center of the circle and any side of the triangle. The distance between the center of the circle and the side lying on the line $x = 8$ can be calculated as follows:

$$r = \frac{|Ax_C + By_C + C|}{\sqrt{A^2 + B^2}} = \frac{|1 \cdot 6{,}5 + 0 \cdot (-5) - 8|}{\sqrt{1^2 + 0^2}} = 1{,}5.$$

The equation of a circle has a standard form:

$$(x - x_C)^2 + (y - y_C)^2 = r^2.$$

Then the equation of the circle inscribed in the given triangle:

$$(x - 6{,}5)^2 + (y + 5)^2 = 1{,}5^2 = 2{,}25.$$

Answer: $(x - 6{,}5)^2 + (y + 5)^2 = 1{,}5^2$.