Answer on Question #58344 – Math – Analytic Geometry

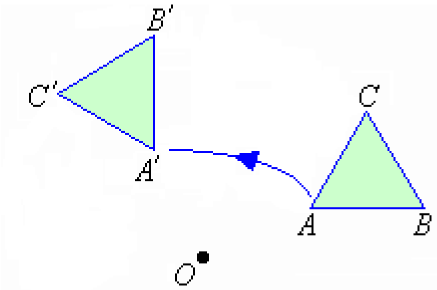
Question

Triangle ABC is rotated to create the image A'B'C'. Which rule describes the transformation?

Solution

In two dimensions (in a plane)

For example, in a plane the triangle *ABC* is rotated about *O* through φ angle in an counterclockwise direction.



Triangle rotated about point O

Usually rotations on a coordinate grid are considered to be counterclockwise, unless otherwise stated.
In analytic geometry a counterclockwise rotation in Cartesian coordinates is expressed by the following formulae:

$$x' = x\cos\varphi - y\sin\varphi$$
$$y' = x\sin\varphi + y\cos\varphi$$

If point A has coordinates (x_A, y_A), then after a counterclockwise rotation trough φ angle the coordinates of point A′ will be

$$x_A' = x_A\cos\varphi - y_A\sin\varphi$$
$$y_A' = x_A\sin\varphi + y_A\cos\varphi$$

Particular cases are as follows:

- Rotation of 0° on coordinate axes

$$x_A' = x_A$$
$$y_A' = y_A$$

- Rotation of 90° on coordinate axes

$$x_A' = -y_A$$
$$y_A' = x_A$$

- Rotation of 180° on coordinate axes (in either direction it is a half-turn)

$$x_A' = -x_A$$
$$y_A' = -y_A$$

- Rotation of 270° on coordinate axes (270° counterclockwise rotation is the same as a 90° clockwise direction)

$$x_A' = y_A$$
$$y_A' = -x_A$$

In analytic geometry a clockwise rotation in Cartesian coordinates is expressed by the following formulae:

$$x' = x\cos\varphi + y\sin\varphi$$
$$y' = -x\sin\varphi + y\cos\varphi$$

In three dimensions (in a space)

If point A has coordinates (x_A, y_A, z_A), then after rotation trough φ angle about the x axis the coordinates of point A′ will be

$$x_A' = x_A$$
$$y_A' = y_A\cos\varphi - z_A\sin\varphi$$
$$z_A' = y_A\sin\varphi + z_A\cos\varphi$$

If point A has coordinates (x_A, y_A, z_A), then after rotation trough φ angle about the y axis the coordinates of point A′ will be

$$x_A' = x_A\cos\varphi + z_A\sin\varphi$$
$$y_A' = y_A$$
$$y_A' = -x_A\sin\varphi + z_A\cos\varphi$$

If point A has coordinates (x_A, y_A, z_A), then after rotation trough φ angle about the z axis the coordinates of point A′ will be

$$x_A' = x_A\cos\varphi - y_A\sin\varphi$$
$$y_A' = x_A\sin\varphi + y_A\cos\varphi$$
$$z_A' = z_A$$

Any other rotation in three-dimensional space can be obtained from these three.

Answer:

Rule which describes the transformation in a plane (where φ is an angle of rotation):

$$x' = x\cos\varphi - y\sin\varphi$$
$$y' = x\sin\varphi + y\cos\varphi$$