Answer on Question #58326 – Math – Other
Question

Maximize the utility function u=x1+x2 subject to y=p1x1+p2x2 also find out demand function.

Solution

Using the Lagrange multiplier obtain

$$L = u(x_1, x_2) + \lambda(y - p_1 x_1 - p_2 x_2)$$

$\frac{\partial L}{\partial x_1} = 0,$

$\frac{\partial L}{\partial x_2} = 0,$

$\frac{\partial L}{\partial \lambda} = 0.$

Hence,

$\frac{\partial u}{\partial x_1} - \lambda p_1 = 0,$

$\frac{\partial u}{\partial x_2} - \lambda p_2 = 0,$

$p_1 x_1 + p_2 x_2 = y.$

In this problem,

$\frac{\partial}{\partial x_1}(x_1 + x_2) - \lambda p_1 = 0,$

$\frac{\partial}{\partial x_2}(x_1 + x_2) - \lambda p_2 = 0,$

$p_1 x_1 + p_2 x_2 = y,$

which is equivalent to

$$1 - \lambda p_1 = 0,$$

$$1 - \lambda p_2 = 0,$$

$$p_1 x_1 + p_2 x_2 = y$$

If $p_1 \neq p_2$ then there is no solution.

If $p_1 = p_2$ then $p_1(x_1 + x_2) = y$, therefore, $u^* = x_1 + x_2 = \frac{y}{p_1}$ is a maximum value of the utility function, here x_1 is any number satisfying $0 \leq x_1 \leq \frac{y}{p_1}$ and $x_2 = \frac{y}{p_1} - x_1$.

The solutions for x_1 and x_2 are called the consumer's demand functions.

Remark. It must be $u = x_1 x_2$ because linear function does not have maximum.

$y = p_1 x_1 + p_2 x_2 \quad \rightarrow \quad x_2 = \frac{y}{p_2} - \frac{p_1}{p_2} x_1$

So $u(x_1) = \frac{y}{p_2} x_1 - \frac{p_1}{p_2} x_1^2$.

Maximum of $u(x_1)$ is attained at the point where $\frac{du}{dx_1} = 0 \ \rightarrow$

$$\rightarrow \frac{y}{p_2} - 2\frac{p_1}{p_2} x_1 = 0 \rightarrow$$

$\rightarrow \quad x_1 = \frac{y}{2p_1}, \quad u_{max} = u\left(\frac{y}{2p_1}\right) = \frac{y}{4p_1 p_2}$.

Demand functions:

$\boldsymbol{x_1 = \frac{y}{2p_1}, \quad x_2 = \frac{y}{2p_2}}$ **.**