

Answer on Question #58300 – Math – Complex Analysis

Question

Express
8i
in polar form

Solution

$z=a+ib=|z|(\cos\varphi+i\sin\varphi)$, where $|z|=\sqrt{a^2+b^2}$ and

$$\varphi = \begin{cases} \arccos\left(\frac{a}{|z|}\right), & b \geq 0, |z| > 0, \\ -\arccos\left(\frac{a}{|z|}\right), & b < 0, |z| > 0, \\ \text{undefined}, & |z| = 0, \end{cases}$$

or

$$\varphi = \begin{cases} \arctan\frac{b}{a}, & a > 0, \\ \frac{\pi}{2}, & a = 0, b > 0, \\ -\frac{\pi}{2}, & a = 0, b < 0, \\ \arctan\frac{b}{a} + \pi, & a < 0, b \geq 0, \\ \arctan\frac{b}{a} - \pi, & a < 0, b < 0. \end{cases}$$

$$z=8i, a = 0, b = 8,$$

$$|z|=\sqrt{0+64}=8; \varphi = \arccos\left(\frac{0}{8}\right) = \frac{\pi}{2}.$$

Then the polar form of $z=8i$ is

$$z=8\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right).$$

Answer: $8i=8\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right).$