Answer on Question #58204 – Math – Differential Equations

Question

1. a) Solve the following ordinary differential equation:

i) 1/x sin ydx + [(ln x)(cos y) + y)] dy = 0

Solution

$$\frac{siny}{x}dx + (lnxcosy + y)dy = 0$$

Let us split the left-hand side of equation into two parts

$$\left(\frac{siny}{x}dx + lnxcosydy\right) + ydy = 0$$

In the first part

$$\frac{siny}{x}dx = sinyd(lnx)$$

and

$$lnxcosydy = lnxd(siny)$$

$$\left(\frac{siny}{x}dx + lnxcosydy\right) + ydy = (siny \cdot d(lnx) + lnx \cdot d(siny)) + ydy$$
$$= d(siny \cdot lnx) + ydy = 0$$

We integrate the last expression and get

$$\boldsymbol{lnx \cdot siny + \frac{y^2}{2} = C,}$$

where C is an arbitrary real constant.

Answer: $\boldsymbol{lnx \cdot siny + \frac{y^2}{2} = C.}$

Question

1. a) Solve the following ordinary differential equation:
ii) d^2 y / dx^2 + dy/dx - 12y = 4e^2x

Solution

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = 4e^{2x}$$

Let us find a solution in the form of

$$y = e^{\mu x}$$

First we solve the homogeneous equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = 0$$

which has the characteristic equation

$$\mu^2 + \mu - 12 = 0$$

Applying Vieta's formula we get $\mu = 3 \ \ or \ \mu = -4$.

Solution to the homogeneous equation is $y_h = Ce^{3x} + De^{-4x}$.
Now, find a particular solution to non-homogeneous equation in the form of

$$y_p = \alpha e^{2x}$$

$$\frac{d^2y_p}{dx^2} + \frac{dy_p}{dx} - 12y_p = 4e^{2x}$$

$$\frac{d^2}{dx^2}(\alpha e^{2x}) + \frac{d}{dx}(\alpha e^{2x}) - 12(\alpha e^{2x}) = 4e^{2x}$$

$$4\alpha e^{2x} + 2\alpha e^{2x} - 12e^{2x} = 4e^{2x}$$

$4\alpha + 2\alpha - 12 = 4 \to 6\alpha = 16 \to \alpha = \frac{16}{6} = \frac{8}{3}$,
hence

$$y_p = \frac{8}{3}e^{2x}$$

and the general solution is

$$y = y_h + y_p = Ce^{3x} + De^{-4x} + \frac{8}{3}e^{2x}$$

Answer: $\boldsymbol{y = Ce^{3x} + De^{-4x} + \frac{8}{3}e^{2x}}$**.**

Question

b) Solve the initial value problem:

d^2 y / dx^2 + 2(dy/dx) + 2y=0, y(0)=2, y'(0)=1

Solution

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

$$y(0) = 2$$
$$y'(0) = 1$$

Let us find a solution in the form of

$$y = e^{\mu x}$$

An homogeneous equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

has a characteristic equation

$$\mu^2 + 2\mu + 2 = 0$$

Applying Vieta's formula we get $\mu = -1 + i \ or \ \mu = -1 - i.$

Solution to homogeneous equation is $y = Ce^{-x}sinx + De^{-x}cosx$
To meet the initial conditions

$$y(0) = 2,$$
$$y'(0) = 1,$$

consider

$$y(0) = 2 = Ce^{-0}sin0 + De^{-0}cos0 = 0 + D = D \to D = 2$$

$$y'(0) = 1 = -Ce^{-0}sin0 + Ce^{-0}cos0 + De^{-0}cos0 - De^{-0}sin0 = 0 + C + D = C + 2 = 1$$
$$\to C = -1$$

Answer: $y = -e^{-x}sinx + 2e^{-x}cosx.$