Answer on Question #58172 – Math – Calculus

Question

Let
y=3x2−2x·4x3−4x2·73x+4

(a) Given that ln(y) can be written in the form
ln(y)=ln(A)x3+ln(B)x2+ln(C)x+ln(D).

What are the coeffcients A, B, C, and D?
A=

B=

C=

D=

Solution

$$y = -8x^4 - 292x^3 + 3x^2 + 4;$$

ln(y)=ln(A)x3+ln(B)x2+ln(C)x+ln(D);

$$y = A^{x^3} \cdot B^{x^2} \cdot C^x \cdot D;$$

If x=0 then y=4, so D=4

If x=1 then y=-293, 4ABC=-293

If x=2 then y=-2448 $A^8B^4C^2 = -612$

If x=3 then y=-8501 $4A^{27}B^9C^3 = -8501$

Thus

> $eqs := \left\{A \cdot B \cdot C = -\frac{293}{4}, A^8 \cdot B^4 \cdot C^2 = -612, A^{27} \cdot B^9 \cdot C^3 = -\frac{8501}{4}\right\};$

$$eqs := \left\{A\,B\,C = -\frac{293}{4}, A^8\,B^4\,C^2 = -612, A^{27}\,B^9\,C^3 = -\frac{8501}{4}\right\}$$

> $a1 := solve(eqs, \{A, B, C\});$

$$a1 := \left\{A = \frac{1}{12}\,RootOf\left(213832088257 + 19652\_Z^6, label = \_L29\right), B = 23970816\,RootOf\left(18357270944775193\_Z^2 - 1, label = \_L45\right), C = \frac{427613869}{6912}\,RootOf\left(18357270944775193\_Z^2 - 1, label = \_L45\right)\,RootOf\left(213832088257 + 19652\_Z^6, label = \_L29\right)^5\right\}$$

>

Question

(b)
Using the correct answer to (a), find the value of dy/dx when x=0.

Solution

Differentiating

ln(y)=ln(A)x3+ln(B)x2+ln(C)x+ln(D)

get

$$\frac{y'}{y} = \ln(A) \cdot 3x^2 + \ln(B) \cdot 2x + \ln(C),$$

hence

$$y' = y\left(3x^2 \ln(A) + 2x\ln(B) + \ln(C)\right).$$

$$y'(0) = y(0)\left(3 \cdot 0^2 \ln(A) + 2 \cdot 0 \cdot \ln(B) + \ln(C)\right) = 4\ln(C).$$