

Answer on Question #58170 – Math – Calculus

Question

If $f(x) = x^3 + 2\cos(x)3\sin(x)$, find $f'(\pi^2)$.

Solution

$$\begin{aligned} f'(x) &= (x^3 + 2\cos(x) \cdot 3\sin(x))' = (x^3)' + (2\cos(x) \cdot 3\sin(x))' = 3x^2 + 6(\cos(x) \cdot \sin(x))' = \\ &= 3x^2 + 6(\cos(x))' \sin(x) + 6(\sin(x))' \cos(x) = 3x^2 - 6\sin(x) \sin(x) + 6\cos(x) \cos(x) = \\ &= 3x^2 + 6(\cos^2(x) - \sin^2(x)); \end{aligned}$$

$$f'(2\pi) = 3 \cdot (2\pi)^2 + 6(\cos^2(2\pi) - \sin^2(2\pi)) = 3 \cdot 4\pi^2 + 6 = 12\pi^2 + 6 \approx 124.435;$$

$$\begin{aligned} f'(\pi^2) &= 3 \cdot (\pi^2)^2 + 6(\cos^2(\pi^2) - \sin^2(\pi^2)) = 3 \cdot \pi^4 + 6\cos(2\pi^2) = 3(\pi^4 + 2\cos(2\pi^2)) \approx \\ &\approx 296.005. \end{aligned}$$

Answer: $f'(2\pi) = 12\pi^2 + 6$, $f'(\pi^2) = 3(\pi^4 + 2\cos(2\pi^2))$.