Answer on Question #58137 – Math – Geometry

1. Water is flowing out of a conical funnel through its apex at a rate of 12 cubic inches per minute. If the funnel is initially full, how long will it take for it to be one-third full? What is the height of the water level? Assume the radius to be 6 inches and the altitude of the cone to be 15 inches.

Solution:

The volume V of the cone is given by $V = \frac{1}{3}Ah$, where $A = \pi r^2$ is the area of the base, h is the altitude of the cone and r is the base radius. Thus $V = \frac{\pi}{3}r^2h \cong 565.487$ cubic inches .

If water is flowing out at the rate of v we obtain:

$$V(t) = V - vt$$

where t is the time of flowing out. Thus in t minutes there remains V_1 cubic inches of water in the funnel.

$$V_1 = \frac{1}{3}V = V - vt$$

Then $t = \frac{2}{3v}V = \frac{2}{3\cdot 12}565.487 \cong 31.416$ minutes or $t \cong 31$ minutes 25 seconds

If the funnel is one-third full we have new values of the volume, base radius r_1 and altitude h_1

$$V \to V_1 = \frac{1}{3}V, r \to r_1, h \to h_1$$

The linear size of the funnel becomes α times smaller: $r = \alpha r_1, h = \alpha h_1$.

So

$$V_1 = \frac{\pi}{3}{r_1}^2 h_1 = \frac{\pi}{3}\frac{r^2}{\alpha^2}\frac{h}{\alpha} = \frac{V}{\alpha^3} = \frac{V}{3}$$

Hence $\alpha^3 = 3$ and the height of the water level is $h_1 = \frac{h}{\sqrt[3]{3}} \cong \frac{15}{1.442} \cong 10.402$ inches.

Answer: It will take about 31 minutes and 26 seconds for funnel to be one-third full. And the height of the water level will be 10.402 inches

2. Two similar cones have volumes 81 pi over 2 inches cube and the slant height of the bigger cone is 7.5 inches. Find the integer solution to the height of the smaller cone.

Solution:

We have $V_b + V_s = \frac{81\pi}{2}, V_s < V_b$

Hence $V_b > \frac{81\pi}{4}$

where V_b is the volume of the bigger cone and V_s is the volume of smaller one:

$V_b = \frac{1}{3}\pi R^2 H$ and $V_s = \frac{1}{3}\pi r^2 n$

R is the radius of the bigger cone, H is its height

r is the radius of the smaller cone, n is integer number, its height

If L is the slant height of the bigger cone, then $R^2 + H^2 = L^2$ and we can write down

$V_b = \frac{1}{3}\pi(L^2 - H^2)H > \frac{81\pi}{4}$

or $(7.5^2 - H^2)H > \frac{243}{4} \Rightarrow (56.25 - H^2)H - 60.75 > 0$

Then $1.105 < H < 6.887$

As $n < H$ then $n \leq 6$.

Maximum of the volume of the bigger cone we can find at H where $\frac{\partial V_b}{\partial H} = 0 \Rightarrow L^2 - 3H^2 = 0$

Then $H = \sqrt{\frac{L^2}{3}} = \sqrt{18.75} \cong 4.33$

and $R = \sqrt{L^2 - H^2} \cong 6.124,\ \frac{R}{H} = 1.414$

Maximum of the volume of the bigger cone is $V_b = \frac{1}{3}\pi(3H^2 - H^2)H = \frac{2}{3}\pi H^3 \cong 54.12\pi > \frac{81\pi}{2}$

Thus minimum of the volume of the smaller cone must be $V_s > 0$

So $n > 0$

Thus $n = 1, 2, 3, 4, 5, 6.$

Answer: The integer solution to the height of the smaller cone can be any integer number from 1 to 6: $n = 1, 2, 3, 4, 5, 6.$

3. If the slant height of the cone is 16 inches and the total area is 120 pi square inches, find the height of the cone.

Solution:

As $A = \pi R^2 + \pi RL$

and $R^2 + H^2 = L^2$

we can write down

$$A = \pi(L^2 - H^2 + \sqrt{L^2 - H^2}L) \Longrightarrow \left(\frac{A}{\pi} - L^2 + H^2\right)^2 = L^2(L^2 - H^2)$$

$$\Longrightarrow (120 - 256 + H^2)^2 = 256(256 - H^2) \Longrightarrow (H^2 - 136)^2 = 65536 - 256H^2)$$

$$) \Longrightarrow H^4 - 16H^2 - 47040 = 0 \Longrightarrow H^2 \cong 225 \Longrightarrow H \cong 15$$

Answer: The height of the cone is about 15 inches

4. What is the height if a right circular cone having a slant height of 6 square root of 10 units and a base radius of 6 units?

Solution:

As $R^2 + H^2 = L^2$

we obtain $H = \sqrt{L^2 - R^2}$

Thus $H = \sqrt{\left(6\sqrt{10}\right)^2 - 6^2} = \sqrt{360 - 36} = 18$ units.

Answer: The height of the right circular cone is 18 units.

5. Find the ratio of the slant height to the radius of a right circular cone in which the volume and lateral area are numerically equal. Assume the altitude of the cone to be 9 units.

Solution:

As $V = \frac{1}{3}\pi R^2 H$, $A_{lat} = \pi RL$ and $V = A_{lat}$

we obtain $\pi R^2 H = 3\pi RL$

Thus $\frac{L}{R} = \frac{H}{3} = 3$ units

Answer: The ratio of the slant height to the radius is 3 units.