Answer on Question #58060 – Math – Linear Algebra

Question

1. Solve the set of linear equations by Gaussian elimination method, find c.

$$\begin{cases} a + 2b + 3c = 5, \\ 3a - b + 2c = 8, \\ 4a - 6b - 4c = -2. \end{cases}$$

Solution

$$\begin{pmatrix} 1 & 2 & 3 & | & 5 \\ 3 & -1 & 2 & | & 8 \\ 4 & -6 & -4 & | & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & | & 5 \\ 0 & -7 & -7 & | & -7 \\ 0 & -14 & -16 & | & -22 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & | & 5 \\ 0 & 1 & 1 & | & 1 \\ 0 & -14 & -16 & | & -22 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & | & 5 \\ 0 & 1 & 1 & | & 1 \\ 0 & 0 & -2 & | & -8 \end{pmatrix}$$

$$\begin{cases} -2c = -8, \\ b + c = 1, \\ a + 2b + 3c = 5; \end{cases} \begin{cases} c = 4, \\ b = -3, \\ a = -1. \end{cases}$$

Answer: 4.

Question

2. Solve the set of linear equations by the matrix method. Solve for b.

$$\begin{cases} a + 3b + 2c = 3, \\ 2a - b - 3c = -8, \\ 5a + 2b + c = 9. \end{cases}$$

Solution

$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{pmatrix} = A; \det A = -1 - 45 + 8 + 10 - 6 + 6 = -28;$$

$A_{11} = -1 + 6 = 5; A_{12} = -(2 + 15) = -17; A_{13} = 4 + 5 = 9; A_{21} = -(3 - 4) = 1; A_{22} = 1 - 10 = -9; A_{23} = -(2 - 15) = 13; A_{31} = -9 + 2 = -7; A_{32} = -(-3 - 4) = 7; A_{33} = -1 - 6 = -7.$

$$A^{-1} = \frac{-1}{28} \begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}.$$

$$\frac{-1}{28} \begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{-1}{28} \begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix} \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix};$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{-1}{28} \begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix} \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix};$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{-1}{28} \begin{pmatrix} -56 \\ 84 \\ -140 \end{pmatrix};$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}.$$

Answer: -3.

Question

3. Solve the set of linear equations by Gaussian elimination method, find b.
$$\begin{cases} a + 2b + 3c = 5, \\ 3a - b + 2c = 8, \\ 4a - 6b - 4c = -2. \end{cases}$$

Solution

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 3 & -1 & 2 & 8 \\ 4 & -6 & -4 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -7 & -7 & -7 \\ 0 & -14 & -16 & -22 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & -14 & -16 & -22 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -8 \end{array}\right)$$

$$\begin{cases} -2c = -8, \\ b + c = 1, \\ a + 2b + 3c = 5; \end{cases} \begin{cases} c = 4, \\ b = -3, \\ a = -1. \end{cases}$$

Answer: -3.

Question

4. Solve the set of linear equations by Gaussian elimination method, find a.
$$\begin{cases} x + 2y + 3z = 5, \\ 3x - y + 2z = 8, \\ 4x - 6y - 4z = -2. \end{cases}$$

Solution

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 3 & -1 & 2 & 8 \\ 4 & -6 & -4 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -7 & -7 & -7 \\ 0 & -14 & -16 & -22 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & -14 & -16 & -22 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -8 \end{array}\right)$$

$$\begin{cases} -2z = -8, \\ y + z = 1, \\ x + 2y + 3z = 5; \end{cases} \begin{cases} z = 4, \\ y = -3, \\ x = -1. \end{cases}$$

Answer: -1.

Question

5. Solve the set of linear equations by the matrix method. Solve for a.
$$\begin{cases} a + 3b + 2c = 3, \\ 2a - b - 3c = -8, \\ 5a + 2b + c = 9. \end{cases}$$

Solution

$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{pmatrix} = A; \det A = -1 - 45 + 8 + 10 - 6 + 6 = -28;$$

$A_{11} = -1 + 6 = 5; A_{12} = -(2 + 15) = -17; A_{13} = 4 + 5 = 9; A_{21} = -(3 - 4) = 1; A_{22} = 1 - 10 = -9; A_{23} = -(2 - 15) = 13; A_{31} = -9 + 2 = -7; A_{32} = -(-3 - 4) = 7; A_{33} = -1 - 6 = -7.$

$$A^{-1} = \frac{-1}{28} \begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}.$$

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$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{-1}{28} \begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix} \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix};$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{-1}{28} \begin{pmatrix} -56 \\ 84 \\ -140 \end{pmatrix};$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}.$$

Answer: 2.

Question

6. For a relation R in A. if , then R is a _____ relation.
transitive
reflexive
symmentric
associative

Answer: the statement of question is not complete.

Question

7. For a relation R in A.[Math Processing Error],it implies that a =b, then R is an _____ relation.
associative
anti- symmetric
complex
transitive

Answer: anti-symmetric.