Answer on Question #58052 – Math – Trigonometry

Question

Prove that $\tan x + \tan 4x + \tan 7x = \tan x \tan 4x \tan 7x$

Solution

$$\tan(a+b+c) = \frac{\tan(a) + \tan(b) + \tan(c) - \tan(a) \cdot \tan(b) \cdot \tan(c)}{1 - (\tan(a) \cdot \tan(b) + \tan(b) \cdot \tan(c) + \tan(c) \cdot \tan(a))}$$

Now consider $a + b + c = n\pi$, then $\tan(a+b+c) = 0$.

So,

$$\tan(a) + \tan(b) + \tan(c) - \tan(a)\tan(b)\tan(c) = 0$$

$$\tan(a) + \tan(b) + \tan(c) = \tan(a)\tan(b)\tan(c)$$

Therefore,

$$x + 4x + 7x = n\pi$$

$$12x = n\pi$$

$$x = \frac{n\pi}{12}.$$