Question

The legs of a right triangle measure 5 inches and 7 units. If θ is the angle between the 5-inch leg and the hypotenuse, cos θ =

A: 0.71
B: 0.07
C: 0.58
D: 0.81

Solution

Hypotenuse is $\sqrt{5^2 + 7^2} = \sqrt{25 + 49} = \sqrt{74}$.

By the definition,

$\cos θ = \frac{5}{\sqrt{5^2+7^2}} = \frac{5}{\sqrt{74}} = 0.581$.

Answer: C: 0.58.

Question

If cos θ = -2/3, which of the following are possible?
Choose all correct answers.

Sinθ = -√5/3 and tanθ = √5/2

sinθ = √5/3 and tanθ = √5/2

cscθ = 3/√5 and tanθ = -√5/2

cscθ = -3/2 and tanθ = √5/2

Solution

If $\cos(\theta) < 0$, then $\frac{\pi}{2} + 2\pi k < \theta < \frac{3\pi}{2} + 2\pi k$, k is integer.

If $\frac{\pi}{2} + 2\pi k < \theta < \pi + 2\pi k$, k is integer, then $\sin(\theta) > 0$,

$\sin(\theta) = \sqrt{1 - \cos^2(\theta)} = \sqrt{1 - \left(-\frac{2}{3}\right)^2} = \sqrt{1 - \frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$,

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = -\frac{\sqrt{5}}{2}$,

$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}}$

If $\pi + 2\pi k < \theta < \frac{3\pi}{2} + 2\pi k$, k is integer, then $\sin(\theta) < 0$,

$$\sin(\theta) = -\sqrt{1 - \cos^2(\theta)} = -\sqrt{1 - \left(-\frac{2}{3}\right)^2} = -\sqrt{1 - \frac{4}{9}} = -\sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3},$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{-\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = \frac{\sqrt{5}}{2},$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{1}{-\frac{\sqrt{5}}{3}} = -\frac{3}{\sqrt{5}}.$$

Answer:
Sinθ = -√5/3 and tanθ = √5/2 ;
cscθ = 3/√5 and tanθ = -√5/2 .