Answer on Question #57880 – Math – Algebra

Question

Using technique of summing an infinite geometric series, find the fractional representation of $2.72727272\ldots$.

Solution

The infinite geometric series is a series in form $\sum_{n=0}^{\infty} aq^n = a + aq + aq^2 + \cdots + aq^n + \cdots$

We know that sum of the infinite geometric series with $|q| < 1$ equals $\sum_{n=0}^{\infty} aq^n = \frac{a}{1-q}$

Rewrite the given number as

$$2.72727272\ldots = 2 + 0.72727272\ldots = 2 + 0.72 + 0.0072 + 0.000072 + \cdots = 2 + \frac{72}{100} + \frac{72}{100^2} + \frac{72}{100^3} + \cdots$$

We obtain the infinite geometric series

$$\frac{72}{100} + \frac{72}{100^2} + \frac{72}{100^3} + \cdots$$

with $a = \frac{72}{100}$ and $q = \frac{1}{100}$.

Then its sum is

$$\frac{72}{100} + \frac{72}{100^2} + \frac{72}{100^3} + \cdots = \frac{\frac{72}{100}}{1 - \frac{1}{100}} = \frac{\frac{72}{100}}{\frac{99}{100}} = \frac{72}{99} = \frac{8}{11}$$

So the given number has the fractional representation

$$2.72727272\ldots = 2\frac{8}{11}$$

The other way is rewriting given number in form

$$2.72727272\ldots = \frac{27}{10} + \frac{27}{1000} + \frac{27}{100000} + \cdots = \frac{27}{10} + \frac{27}{10}\frac{1}{100} + \frac{27}{10}\frac{1}{100^2} + \cdots$$

This is an infinite geometric series with $a = \frac{27}{10}$ and $q = \frac{1}{100}$.

Then

$$2.72727272\ldots = \frac{\frac{27}{10}}{1 - \frac{1}{100}} = \frac{\frac{270}{100}}{\frac{99}{100}} = \frac{270}{99} = \frac{30}{11} = 2\frac{8}{11}$$

Answer: $2.72727272\ldots = 2\frac{8}{11}$.