Answer on Question #57676 – Math – Analytic Geometry

Question

Find the equation of the circle determined by the given condition:
10. tangent to the x-axis and passing through $(5,1)$ and $(12,8)$.

Solution

Equation of circle $(x-a)^2+(y-b)^2=R^2$, where (a,b) is the center of the circle, R is the radius of the circle. Hence

$$\begin{cases} (5-a)^2+(1-b)^2=R^2 \\ (12-a)^2+(8-b)^2=R^2 \\ |b|=R \end{cases}$$

Solutions of the system are $a_1=0, b_1=13$ and $a_2=8, b_2=5$.
Equations of circle are $x^2+(y-13)^2=13^2$ and $(x-8)^2+(y-5)^2=5^2$.

Answer: $x^2+(y-13)^2=13^2,\ (x-8)^2+(y-5)^2=5^2$.

Question

Find the equation of the circle determined by the given condition:
11. tangent to the line $y=-1$ and containing the point $(1,1)$ and $(3,3)$.

Solution

Equation of the circle is $(x-a)^2+(y-b)^2=R^2$, where (a,b) is the center of the circle, R is the radius of the circle. Hence

$$\begin{cases} (1-a)^2+(1-b)^2=R^2 \\ (3-a)^2+(3-b)^2=R^2 \\ |b+1|=R \end{cases}$$

Solutions of the system are $a_1=3, b_1=1$ and $a_2=-5, b_2=9$.
Equations of circle are $(x-3)^2+(y-1)^2=2^2$ and $(x+5)^2+(y-9)^2=10^2$.

Answer: $(x-3)^2+(y-1)^2=2^2,\ (x+5)^2+(y-9)^2=10^2$.

Question

Find the equation of the circle determined by the given condition:
12. tangent to the line $2x+y=4$, $2x+y=2$ and $x-2y+5=0$.

Solution

Equation of circle is $(x-a)^2+(y-b)^2=R^2$, where (a,b) is the center of the circle, R is the radius of the circle. Hence

$$\begin{cases} \frac{|2a+b-4|}{\sqrt{5}}=R \\ \frac{|2a+b-2|}{\sqrt{5}}=R \\ \frac{|a-2b+5|}{\sqrt{5}}=R \end{cases}$$

Solutions of the system are $a_1=\frac{2}{5}, b_1=\frac{11}{5}$ and $a_2=0, b_2=3$.

Equations of the circle are $(x-\frac{2}{5})^2+\left(y-\frac{11}{5}\right)^2=\frac{1}{5}$ and $x^2+(y-3)^2=\frac{1}{5}$.

Answer: $(x-\frac{2}{5})^2+\left(y-\frac{11}{5}\right)^2=\frac{1}{5},\ x^2+(y-3)^2=\frac{1}{5}$.

Question

Find the equation of the circle determined by the given condition:

13. passing through the origin and tangent to the line $3x + 4y - 10 = 0$, $4x + 3y - 5 = 0$

Solution

Equation of circle $(x-a)^2 + (y-b)^2 = R^2$, where (a, b) is the center of the circle, R is the radius of the circle. Hence

$$\begin{cases} a^2 + b^2 = R^2 \\ \frac{|3a+4b-10|}{5} = R \\ \frac{|4a+3b-5|}{5} = R \end{cases}$$

The system of equations has no solution. Thus, such a circle does not exist.

Answer: Circle does not exist.

Question

Find the equation of the circle determined by the given condition:

14. inscribed in a triangle with sides on the lines $x - 3y = -5$, $3x + y = 1$, $3x - y = -11$.

Solution

Equation of circle $(x-a)^2 + (y-b)^2 = R^2$, where (a, b) is the center of the circle, R is the radius of the circle. Hence

$$\begin{cases} \frac{|a-3b+5|}{\sqrt{10}} = R \\ \frac{|3a+b-1|}{\sqrt{10}} = R \\ \frac{|3a-b+11|}{\sqrt{10}} = R \end{cases}$$

Solution of the system is $a = \frac{-5}{3}, b = \frac{7}{3}$. Equation of the circle is $(x + \frac{5}{3})^2 + \left(y - \frac{7}{3}\right)^2 = \frac{121}{90}$.

Answer: $(x + \frac{5}{3})^2 + \left(y - \frac{7}{3}\right)^2 = \frac{121}{90}$.

Question

Find the equation of the circle determined by the given condition:

15. inscribed in a triangle with vertices (0,6), (8,6), (0,0).

Solution

Equation of the circle is $(x-a)^2 + (y-b)^2 = R^2$, where (a, b) is the center of the circle, R is the radius of the circle. The equations of lines passing through the vertices of the triangle are x=0, y=6, 6x-8y=0. Hence

$$\begin{cases} \frac{|a|}{1} = R \\ \frac{|6-b|}{1} = R \\ \frac{|6a-8b|}{10} = R \end{cases}$$

Solution of the system is $a = 2, b = 4$. Equation of the circle is $(x-2)^2 + (y-4)^2 = 4$.

Answer: $(x-2)^2 + (y-4)^2 = 4$.

Question

Find the equation of the circle determined by the given condition:
16. having radius of square root of 5, through (0,4) and (3,7).

Solution

Equation of the circle is $(x-a)^2+(y-b)^2=R^2$, where (a,b) is the center of the circle, R is the radius of the circle. Hence

$$\begin{cases} a^2+(4-b)^2=5 \\ (3-a)^2+(7-b)^2=5 \end{cases}$$

Solutions of the system are $a_1=1, b_1=6$ and $a_2=2, b_2=5$.
Equations of the circle are $(x-1)^2+(y-6)^2=5$ and $(x-2)^2+(y-5)^2=5$.

Answer: $(x-1)^2+(y-6)^2=5,\ (x-2)^2+(y-5)^2=5$.

Question

Find the equation of the circle determined by the given condition:
17. radius and tangent to the line 2x+y-1=0 at (1,-1).

Solution

Equation of the circle is $(x-a)^2+(y-b)^2=R^2$, where (a,b) is the center of the circle, R is the radius of the circle.
Line perpendicular to 2x+y-1=0 at (1, -1) is $\frac{x-1}{2}=\frac{y+1}{1}$, hence x-2y-3=0 and *a*-2*b*-3=0, that is, *a*=2*b*+3.
Given $R=\sqrt{(a-1)^2+(b+1)^2}$, hence
$R=\sqrt{(2b+3-1)^2+(b+1)^2}=\sqrt{(2b+2)^2+(b+1)^2}=\sqrt{5b^2+10b+5}=\sqrt{5}|b+1|$, that is, $|b+1|=\frac{R}{\sqrt{5}}$.
Thus, $b_1=-1-\frac{R}{\sqrt{5}}$ or $b_2=-1+\frac{R}{\sqrt{5}}$, hence $a_1=2b_1+3=2\left(-1-\frac{R}{\sqrt{5}}\right)+3=1-\frac{2R}{\sqrt{5}}$ or $a_2=2b_2+3=2\left(-1+\frac{R}{\sqrt{5}}\right)+3=1+\frac{2R}{\sqrt{5}}$.
Equations of the circle are $(x-1+\frac{2R}{\sqrt{5}})^2+\left(y+1+\frac{R}{\sqrt{5}}\right)^2=R^2$ and $(x-1-\frac{2R}{\sqrt{5}})^2+\left(y+1-\frac{R}{\sqrt{5}}\right)^2=R^2$.

Answer: $(x-1+\frac{2R}{\sqrt{5}})^2+\left(y+1+\frac{R}{\sqrt{5}}\right)^2=R^2,\ (x-1-\frac{2R}{\sqrt{5}})^2+\left(y+1-\frac{R}{\sqrt{5}}\right)^2=R^2$.

Question

Find the equation of the circle determined by the given condition:

18. tangent to 3x-2y-9=0 and 2x-3y-1=0 and center on 2x+y-10=0.

Solution

Equation of circle is $(x-a)^2+(y-b)^2=R^2$, where (a,b) is the center of the circle, R is the radius of the circle.

$$\begin{cases} 2a + b - 10 = 0 \\ \frac{|3a-2b-9|}{\sqrt{13}} = R \\ \frac{|2a-3b-1|}{\sqrt{13}} = R \end{cases}$$

Solutions of the system are $a_1 = 2$, $b_1 = 6$ and $a_2 = 4$, $b_2 = 2$.

Equations of the circle are $(x-2)^2 + (y-6)^2 = \frac{225}{13}$ and $(x-4)^2 + (y-2)^2 = 1$.

Answer: $(x-2)^2 + (y-6)^2 = \frac{225}{13}$, $(x-4)^2 + (y-2)^2 = 1$.