Answer the question #57675 – Mathematics – Analytic Geometry

Find the equation of the circle determined by the given conditions:

1. passing through $(1, 2)$, $(2, 3)$ and $(-2, 1)$
2. passing through $(4, 6)$, $(-2, -2)$ and $(-4, 2)$
3. circumscribing a triangle formed by the lines $x - y + 2 = 0$, $2x + 3y - 1 = 0$ and $4x + y + 17 = 0$
4. center of the x-axis and passing through $(0, 0)$ and $(2, 4)$
5. containing the point $(2, 2)$ and tangent to the lines $y = 1$ and $y = 6$.
6. tangent to the x-axis and passing through $(5, 1)$ and $(-2, 8)$
7. passing through $(-3, -1)$, $(3, -5)$ and having its center on the line $2x - y - 2 = 0$
8. tangent to the line $4x - 3y = 6$ at $(3, 2)$ and passing through $(2, -1)$
9. tangent to the line $3x - 4y - 5 = 0$ at $(3, 1)$ and passing through $(-3, -1)$

Solutions.

1. So as an equation of any circle is $(x-a)^2 + (y-b)^2 = r^2$ then we have the next system:

$$\begin{cases} (1-a)^2 + (2-b)^2 = r^2, \\ (2-a)^2 + (3-b)^2 = r^2, \\ (-2-a)^2 + (1-b)^2 = r^2, \end{cases} \to \begin{cases} 1 - 2a + a^2 + 4 - 4b + b^2 = r^2, \\ 4 - 4a + a^2 + 9 - 6b + b^2 = r^2, \\ 4 + 4a + a^2 + 1 - 2b + b^2 = r^2, \end{cases} \to$$

$$\begin{cases} 2a + 2b = 8, \\ 6a + 2b = 0, \\ 5 - 2a + a^2 - 4b + b^2 = r^2, \end{cases} \to \begin{cases} a = -2, \\ b = 6, \\ r^2 = 25. \end{cases}$$

Thus, the equation of the circle determined by the given conditions is $\underline{(x+2)^2 + (y-6)^2 = 25}$.

2. So as an equation of any circle is $(x-a)^2 + (y-b)^2 = r^2$ then we have the next system:

$$\begin{cases} (4-a)^2 + (6-b)^2 = r^2, \\ (-2-a)^2 + (-2-b)^2 = r^2, \\ (-4-a)^2 + (2-b)^2 = r^2, \end{cases} \to \begin{cases} 16 - 8a + a^2 + 36 - 12b + b^2 = r^2, \\ 4 + 4a + a^2 + 4 + 4b + b^2 = r^2, \\ 16 + 8a + a^2 + 4 - 4b + b^2 = r^2, \end{cases} \to$$

$$\begin{cases} 12a + 16b = 44, \\ 4a - 8b = -12, \\ 8 + 4a + a^2 + 4b + b^2 = r^2, \end{cases} \to \begin{cases} 3a + 4b = 11, \\ 4b - 2a = 6, \\ 8 + 4a + a^2 + 4b + b^2 = r^2, \end{cases} \to \begin{cases} a = 1, \\ b = 2, \\ r^2 = 25. \end{cases}$$

Thus, the equation of the circle determined by the given conditions is $\underline{(x-1)^2 + (y-2)^2 = 25}$.

3. The first we solve the systems to find three point of the circle:

$\begin{cases} x - y + 2 = 0, \\ 2x + 3y - 1 = 0, \end{cases} \begin{cases} x = y - 2, \\ 5y - 5 = 0, \end{cases} \begin{cases} x = -1, \\ y = 1, \end{cases}$ We get the point $\underline{(-1, 1)}$; $\begin{cases} x - y + 2 = 0, \\ 4x + y + 17 = 0, \end{cases}$ $\begin{cases} x = y - 2, \\ 5y = 9, \end{cases} \begin{cases} x = -3.8, \\ y = -1.8, \end{cases}$ we get the point $\underline{(-3.8, -1.8)}$; $\begin{cases} 4x + y + 17 = 0, \\ 2x + 3y - 1 = 0, \end{cases} \begin{cases} 10x = -52, \\ 5y = 19, \end{cases}$ $\begin{cases} x = -5.2, \\ y = 1.8, \end{cases}$ we get the point $\underline{(-5.2, 3.8)}$.

So as an equation of any circle is $(x-a)^2 + (y-b)^2 = r^2$ then we have the next system:

$$\begin{cases} (-1-a)^2 + (1-b)^2 = r^2, \\ (-3.8-a)^2 + (-1.8-b)^2 = r^2, \\ (-5.2-a)^2 + (3.8-b)^2 = r^2, \end{cases} \to \begin{cases} 1 + 2a + a^2 + 1 - 2b + b^2 = r^2, \\ 14.44 + 7.6a + a^2 + 3.24 + 3.6b + b^2 = r^2, \\ 27.04 + 10.4a + a^2 + 14.44 - 7.6b + b^2 = r^2, \end{cases} \to$$

$$\begin{cases} 5.6a + 5.6b = -15.68, \\ 2.8a - 11.2b = -23.8, \\ 2 + 2a + a^2 + 2b + b^2 = r^2, \end{cases} \to \begin{cases} a = -3.94, \\ b = 1.14, \\ r^2 = 8.6632. \end{cases}$$

Thus, the equation of the circle determined by the given conditions is

$\underline{(x+3.94)^2 + (y-1.14)^2 = 8.6632}$.

4. The line, which is passing through $0,0)$ and $B(2,4)$ is $\frac{x}{2}=\frac{y}{4}$ or $2x-y=0$. The midpoint of the segment AB is $M(1,2)$ and midperpendicular to the line AB is $x-1+2(y-2)=0$ or $x+2y-5=0$ and if the center $a,b)$ of the circle $x-a)^2+\ y-b)^2=r^2$ is on the x-axis then the center is the point (5, 0) and its equation is $x-5)^2+y^2=25$.

5. If the circle tangent to the lines $y=1$ and $y=6$, then its center is on the line $y=\frac{1+6}{2}=3.5$ and its radius is equal to 2.5. So as the circle is containing the point $2,2)$ then we have: $2-a)^2+\ 2-3.5)^2=2.5^2$; $2-a)^2=6.25-2.25=4$; $2-a=\pm 2$ and $a=0$ or $a=4$. Thus, the equation of the circle determined by the given conditions is $\underline{x-4)^2+\ y-3.5)^2=625}$ or $\underline{x^2+(y-3.5)^2=625}$.

6.If the circle is passing through the points $5,1)$ and $B(-2,8)$ then its center is on the line, which is midperpendicular to the line AB: $\frac{x+2}{7}=\frac{y-8}{-7}$ or $x+y-6=0$. The midpoint of AB is $M\ 1.5,4.5)$, then he center is on the line $(x-1.5)-y+4.5=0$ or $x-y+3=0$. In other side, if circle tangent to the x-axis the center $a,b)$ of this circle must satisfies the condition $b^2=\ a-5)^2+\ b-1)^2$. We have the next system:

$$\begin{cases} a-b+3=0, \\ b^2=\ a-5)^2+\ b-1)^2, \end{cases} \rightarrow \begin{cases} b=a+3, \\ b^2=a^2-10a+25+b^2-2b+1, \end{cases} \rightarrow$$

$$\begin{cases} b=a+3, \\ a^2-10a+26-2b=0, \end{cases} \rightarrow \begin{cases} b=a+3, \\ a^2-12a+20=0, \end{cases} \rightarrow \begin{cases} b=5, \\ a=2. \end{cases}$$

and the equation of the circle determined by the given conditions is $\underline{(x-2)^2+\ y-5)^2=25}$.

7. If the circle is passing through the points $-3,-1)$ and $B(3,-5)$ then its center is on the line, which is midperpendicular to the line AB: $\frac{x+3}{6}=\frac{y+1}{-4}$ or $2x+3y+9=0$. The midpoint of AB is $M(0,-3)$, then he center is on the line $3x-2(y+3)=0$ or $3x-2y-6=0$. In other side, the center $a,b)$ of this circle is the line $2x-y-2=0$. We have the next system:

$$\begin{cases} 3a-2b-6=0, \\ 2a-b-2=0, \end{cases} \rightarrow \begin{cases} a=-2, \\ b=-6. \end{cases}$$

The square of radius of this circle is equal to $r^2=\ -3+2)^2+\ -1+6)^2=26$ and the equation of the circle determined by the given conditions is $\underline{(x+2)^2+\ y+6)^2=26}$.

8. If the circle is tangent to the line $4x-3y=6$ at $3,2)$ then its center is on the line, which is perpendicular to the given line and is containing the point $3,2)$. So it is line $3(x-3)+4(y-2)=0$ or $3x+4y-17=0$. In other side, if circle is passing through $2,-1)$ then the center $a,b)$ of this circle must satisfy the condition $a-3)^2+\ b-2)^2=\ a-2)^2+\ b+1)^2$. We have the next system:

$$\begin{cases} 3a+4b-17=0, \\ a-3)^2+\ b-2)^2=\ a-2)^2+\ b+1)^2, \end{cases} \rightarrow \begin{cases} 3a=17-4b, \\ -6a+9-4b+4=-4a+4+2b+1, \end{cases} \rightarrow$$

$$\begin{cases} 3a=17-4b, \\ a=4-3b, \end{cases} \rightarrow \begin{cases} b=-1, \\ a=7. \end{cases}$$

and the equation of the circle determined by the given conditions is $\underline{(x-7)^2+\ y+1)^2=25}$.

9. If the circle is tangent to the line $3x-4y-5=0$ at $3,1)$ then its center is on the line, which is perpendicular to the given line and is containing the point $3,1)$. So it is line $4(x-3)+3\ y-1)=0$ or $4x+3y-15=0$. In other side, if circle is passing through $-3,-1)$ then the center $a,b)$ of this circle must satisfy the condition $a-3)^2+\ b-1)^2=\ a+3)^2+\ b+1)^2$. We have the next system:

$$\begin{cases} 4a+3b-15=0, \\ a-3)^2+\ b-1)^2=\ a+3)^2+\ b+1)^2, \end{cases} \rightarrow \begin{cases} 4a=15-3b, \\ -6a+9-2b+1=6a+9+2b+1, \end{cases} \rightarrow$$

$$\begin{cases} 4a=15-3b, \\ 12a=-4b, \end{cases} \rightarrow \begin{cases} b=9, \\ a=-3. \end{cases}$$

and the equation of the circle determined by the given conditions is $\underline{(x+3)^2+\ y-9)^2=100}$.

Answer: all sections are solved.