Question #57674

The problem is:

Write the equation of the circle satisfying the given equations:
1. with center (2, -1) and radius 3.
2. with center (1, 0) and radius square root of 5 over 3
3. with center (0, 0) and diameter 8.
4.with center (3, 4) and passing through (-1, 1).
5. center at (-3, 2) and touching the x-axis
6. center at (4, 1) touching the y-axis.
7. with radius 3, touching both axes and center in 3rd quadrant.
8. with radius 2, tangent to both axes and center in 4th quadrant.
9. with center at (0, 3) and touching the line x - y = 0
10. with center at (-2, -1) and tangent to the line 4x - 3y = 12.
11. with center at (3,2) and tangent to the line passing through (-1, 3) and (-4,0)

Solve:

The equation of the circle looks like $(x-a)^2+(y-b)^2=R^2$, (1)

where a, b - the coordinates ofthe circlecenter, R - the radius.

1. The circlewith center (2, -1) and radius 3.
   We have $a=2,\ b=-1,\ R=3$
   Using (1): $(x-2)^2+(y-(-1))^2=3^2$ or $(x-2)^2+(y+1)^2=9$
2. The circlewith center (1, 0) and radius square root of 5 over 3
   $a=1,\ b=0,\ R=\sqrt{\frac{5}{3}}$
   Using (1): $(x-1)^2+y^2=\left(\sqrt{\frac{5}{3}}\right)^2=\frac{5}{3}$.
3. The circlewith center (0, 0) and diameter 8.
   $a=0,\ b=0,\ R=\frac{diameter}{2}=\frac{8}{2}=4$
   Using (1): $x^2+y^2=4^2=16$
4. The circlewith center (3, 4) and passing through (-1, 1).
   $a=3,\ b=4$, but we don't know R.
   In generalthe equation of the circle is $(x-3)^2+(y-4)^2=R^2$
   But the circlepasses through(-1, 1), and thenpoint lieson the circlesatisfies the equation $(x-3)^2+(y-4)^2=R^2$.
   So $((-1)-3)^2+(1-4)^2=R^2$
   $(-4)^2+(-3)^2=R^2$
   $25=R^2$
   $R=5$
   And now the equation of the circle is $(x-3)^2+(y-4)^2=5^2=25$

5. The circlewith center at (-3, 2) and touching the x-axis.

   $a=-3,\ b=2$

   If circle is touching the x-axis it's means, that module of y-coordinate of the center is radius of this circle.

   $R=|b|=2$

   $(x-(-3))^2+(y-2)^2=2^2=4$

   $(x+3)^2+(y-2)^2=4$

6. The circlewithcenter at (4, 1) touching the y-axis.

   $a=4,\ b=1$

   If circle is touching the y-axis it's means, that module of x-coordinate of the center is radius of this circle.

   $R=|a|=4$

   $(x-4)^2+(y-1)^2=4^2=16$

7. The circlewith radius 3, touching both axes and center in 3rd quadrant.

   If the circle is touching both axes his center in the point $(\pm R;\pm R)$.

   Using condition that center in 3rd quadrant: $a<0,\ b<0$.

   So we have circle with radius 3 with center (-3,-3).

   Using (1): $(x+3)^2+(y+3)^2=3^2=9$

8. The circlewith radius 2, tangent to both axes and center in 4th quadrant

   If the circle is touching both axes his center in the point $(\pm R;\pm R)$.

   Using condition that center in 4th quadrant: $a>0,\ b<0$.

   So we have circle with radius 3 with center (2,-2).

   Using (1): $(x-2)^2+(y+2)^2=2^2=4$

9. The circlewith center at (0, 3) and touching the line x - y = 0

   The radius of this circle = distance from the center and the line x-y=0.

   If you specifythe equation ofthe lineAx + By + C =0, then the distance fromM (a, b)point to linecan be foundusing the following formula

   $$d=\frac{|Aa+Bb+C|}{\sqrt{A^2+B^2}}$$

   In our case A=1, B=-1, C=0, a=0, b=3

   $$d=R=\frac{|1\cdot 0-1\cdot 3+0|}{\sqrt{1^2+(-1)^2}}=\frac{|-2|}{\sqrt{2}}=\sqrt{2}$$

   Using (1): $x^2+(y-3)^2=\left(\sqrt{2}\right)^2=2$

10. The circlewith center at (-2, -1) and tangent to the line 4x - 3y = 12.

    A=4, B=-3, C=-12, a=-2, b=-1

    $$d=R=\frac{|4\cdot(-2)+(-3)\cdot(-1)-12|}{\sqrt{4^2+(-3)^2}}=\frac{20}{5}=4$$

    Using (1): $(x+2)^2+(y+1)^2=4^2=16$

11. The circle with center at (3,2) and tangent to the line passing through (-1, 3) and (-4,0).

Canonical equation ofa line passingthrough two pointswithcoordinates (x_1, y_1), (x_2, y_2) looks likeformula $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$.

$(x_1, y_1)=(-1,3)$

$(x_2, y_2)=(-4,0)$

The line is: $\frac{x+1}{-4+1}=\frac{y-3}{0-3}$

$\frac{x+1}{-3}=\frac{y-3}{-3}$

$x+1=y-3$

Line is $x-y+4=0$

A=1, B=-1, C=4, a=3, b=2

$d=R=\frac{|1\cdot 3+(-1)\cdot 2+4|}{\sqrt{1^2+(-1)^2}}=\frac{5}{\sqrt{2}}$

Using (1): $(x-3)^2+(y-2)^2=\left(\frac{5}{\sqrt{2}}\right)^2=\frac{25}{2}$

Answers:

1. $(x-2)^2+(y+1)^2=9$
2. $(x-1)^2+y^2=\frac{5}{3}$
3. $x^2+y^2=16$
4. $(x-3)^2+(y-4)^2=25$
5. $(x+3)^2+(y-2)^2=4$
6. $(x-4)^2+(y-1)^2=16$
7. $(x+3)^2+(y+3)^2=9$
8. $(x-2)^2+(y+2)^2=4$
9. $x^2+(y-3)^2=2$
10. $(x+2)^2+(y+1)^2=16$
11. $(x-3)^2+(y-2)^2=\frac{25}{2}$